Hands-On Math
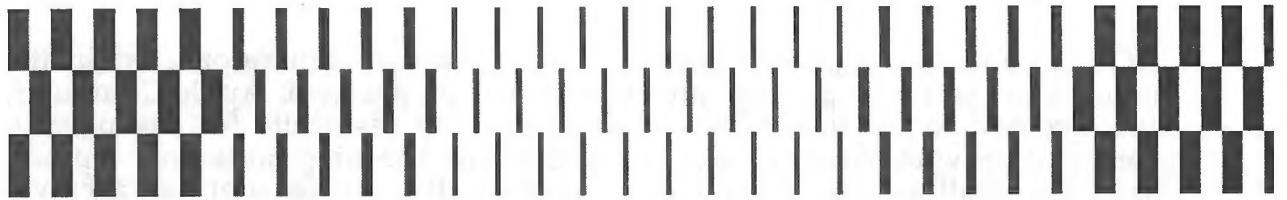
Volume 2

Computer Simulated
Math Manipulatives

For Apple //e, //c or //GS

Ventura Educational Systems

Hands-On Math: Volume II
Learning with Computers and Math Manipulatives

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Jonathan Ventura is entering Kindergarten in the Fall of 1988. He has had five years of experience with computers and has been very helpful in the initial testing of these programs.

Other Publications Include:

SuperGraph	Coordinate Geometry
GeoArt: Geometry and Art Discovery Unit	Geometry Concepts
Marine Life: Anatomy of a Fish	Marine Invertebrates
Anatomy of a Sea Lamprey	Anatomy of a Shark
Senses: Physiology of the Human Sense Organs	Hands-On Math Vol. I
The Plant: Nature's Food Factory	Hands-On Math Vol. III
Chemaid: Introduction to the Periodic Table	Beginning Geometry
The Worm: Invertebrate Anatomy	Computer Concepts
Protozoa: Introduction to Microorganisms	Plant and Animal Cells
States: Geography Study Unit and Database	The Insect World
All About the Solar System	All About Matter
All About Simple Machines	All About Light & Sound
Dr. Know: Experiments in Artificial Intelligence	Algebra Concepts
Clip-Art for Science Teachers	Clip-Art for Math Teachers
VisiFrog: Vertebrate Anatomy	Computer Concepts
Music Concepts	Plant and Animal Cells

Additional Program Disks

Many schools have more than one computer and to effectively use educational software require additional legal copies of a program. Additional program disks are available for use in a computer lab. The price is $10.00 per disk. Schools with a registered copy of any Ventura Educational Systems product may order additional copies of a program disk at any time. There is a 30 day warranty on original program disks. If for any reason a program disk becomes defective within 30 days of the date of purchase, Ventura Educational Systems will replace it at no charge.

Special Thanks

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Overview

Children learn best through active involvement in the learning process. This program is designed to be a tool that teachers can use for active learning. Manipulative devices can be a rich source of teaching strategies for problem solving and can be very helpful in developing intuitive mathematical concepts. The Hands-On Math Series suggests ways in which concrete learning experiences can be extended to a representation level and still remain manipulative. All of these strategies are aimed at the eventual goal of working with mathematics at an abstract level.

This manual consists of two sections. The first part is written for the teacher and explains the menu options available in each program and also presents ideas for instructional strategies that can be implemented with each simulated manipulative device. The second section of the manual is a set of reproducible activity sheets that are designed to assist children in using the Hands-On Math program. The activities have been developed for elementary age children and are arranged in order by manipulative device. Teachers will want to decide what is the best sequence for using the materials. Each lesson is aimed at specific mathematical objectives including addition, subtraction, multiplication, division, place value, fractions, probability, geometry, and logic. Each activity is meant to be a beginning. Teachers will want to encourage the children to explore extensions of each activity. Orally discussing each activity will help to foster higher level thinking.

Hands-On Math is a starting point. Learning should be fun and as students work with the program it is my intention that they will begin discussing, sharing and creatively exploring mathematics.

-- Fred Ventura, Ph.D.

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Using a Computer in a Manipulative Approach to Math

Approaches to the teaching of mathematics that rely heavily on one methodology are inherently weak and unlikely to produce optimal results. Educators have found that teaching strategies must adapt to accommodate new discoveries which expand our understanding of the learning process and new technologies which expand our delivery systems.

According to current learning theory, children learn best when they are actively involved in the learning process. There are many ways to do this but one example is having children work in small groups in a laboratory/discovery situation. Small group instruction encourages variation in teaching methodology. Varying the way in which material is presented serves the instructional process since one particular methodology may not be best for all children. Different children respond differently to a particular educational approach. The same methodology that is appropriate for one content area may not be as effective with a different content area.

For learning mathematics an active teaching and active learning situation is a very desirable educational environment. To create it the teacher must be aware of the behavioral characteristics of the students with regard to mathematics, must be knowledgeable in the particular skills which are being taught and must be able to draw upon diverse strategies in order to decide which is the most appropriate for fostering the development of the targeted mathematical concepts.

In general, educational psychologists believe that the ability of children to learn passes through developmental stages. Each stage is characterized by particular behaviors. In the early stages learning is tied to perceptual responses. As the child matures abstract reasoning becomes possible. Concrete models are useful for laying the conceptual groundwork for new ideas but once a concept has been internalized the concrete models are no longer necessary. The work of the Swiss psychologist, Jean Piaget, has contributed a great deal of support to this theory, and has fostered the development of new educational strategies which are consistent with the theory.

Introduction to Hands-On Math

The Hands-On Math series combines the use of concrete materials for teaching mathematics with the use of the computer. When used in conjunction with actual manipulative devices the programs offer a unique set of strategies for active learning. While using this program students can draw upon concepts developed from concrete experiences using manipulative devices and will work with the same concepts in a representational manner using the computer. In this

strategy the child's <u>concrete</u> mathematical knowledge is used in a transitional way as the foundation for the development of <u>abstract</u> mathematical thinking skills. Once mathematical concepts have been internalized by the child in concrete and representational ways, the stage is set for an understanding of the more formal, abstract axioms of higher mathematics.

Hands-On Math, Volume II simulates the use of five manipulative devices: Two Color Counters, Color Tiles, Mirrors, Attribute Blocks, and Base Ten Blocks. For each device a program called the Playground provides the child with a free-form work area. Using the Playground students can move objects on the screen and experiment with mathematical ideas. The Playground is also used with activity pages in order to present mathematical concepts in a structured way. In addition to the Playground a variety of other exercises can be selected for each manipulative. Reproducible activity pages are provided to guide students through the use of each program and as examples of the learning tasks that students can do with the program. The examples in this teacher's guide are designed to suggest ways in which Hands-On Math can be integrated with the traditional curriculum.

Hardware Requirements

Hands-On Math is designed to work with most standard Apple computer systems. The hardware requirements are listed below:

> Apple //e, Apple //c or Apple//gs Computer System
> 64K RAM (minimum)
> Single or Dual Disk
> Video Monitor (Color recommended)
> DOS 3.3/Applesoft BASIC and Assembler
> Storage Disk (DOS 3.3 format)

Getting Started

Before you begin using Hands-On Math, be sure you have ordered or made your backup of the program disk and the data disk. Use the copy and store the original in a safe place. The purchaser of this program is entitled to keep one backup of this program for archival purposes. (Using the backup in a second computer or making more than one copy is considered a copyright infringement. Additional copies of the program disk are available and may be purchased by schools with computer labs.)

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Start Hands-On Math with the program disk in Drive 1. Power ON the system or restart by pressing CTRL-Open Apple-Reset or use PR#6 to warm-start the systems. Please consult your Apple User's Guide for more complete system operating instructions.

Menu System

Hands-On Math encourages exploration. The program is designed in such a way that the physical operation of the computer does not interfere with the learning activity. Control over the program is exercised by the use of four keys. Each key has a consistent function throughout the program. *Apple // plus users can press control -J for down and control-I for up.*
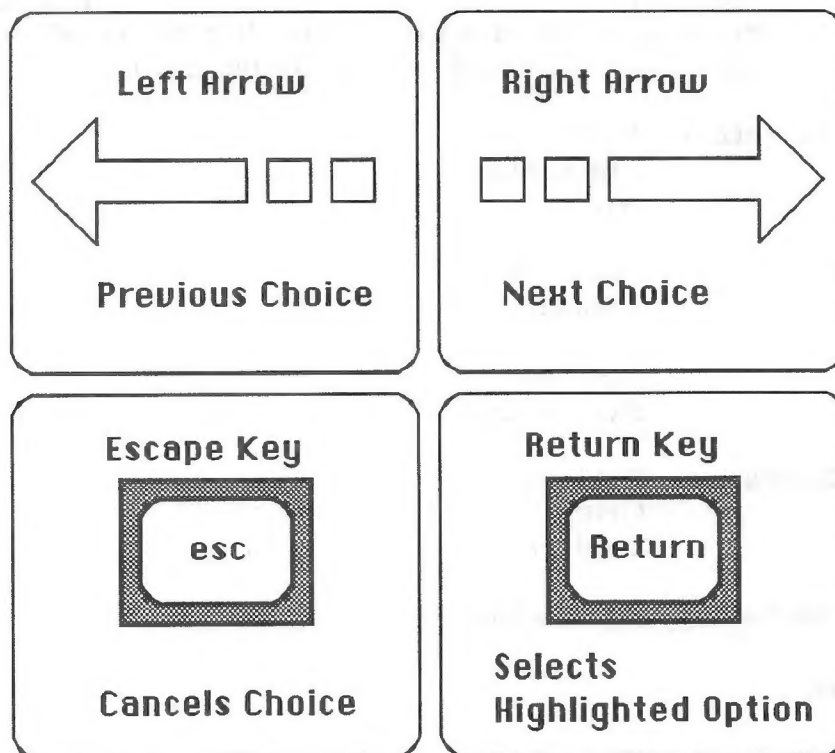
When using Hands-On Math, a menu bar is displayed at the top of the screen. The options in each menu are selected by pressing the right or left arrow keys to highlight an item. The return key activates the highlighted option. Whenever appropriate, the escape key can be used to cancel a choice. Choosing escape exits the current menu and returns control to a higher level menu. When the program has been properly started, the menu shown below will appear on the screen.

Playground Exercises Configuration Exit

Playground

The Playground is where students can 'play' with mathematical ideas. Choosing this option leads to a second level menu where a manipulative device is selected. The manipulatives available in Hands-On Math Volume II are Two Color Counters, Color Tiles, Mirrors, Attribute Blocks, and Base Ten Blocks. Each of the Playground programs are described individually and in more detail in the section following the general explanation of the main menu.

Exercises

The Exercise option on the main menu also leads to a second level menu where a manipulative device can be selected. Each manipulative has different exercises associated with it. The list below shows the names of the exercises related to each manipulative device.

Two Color Counters	Skills
	Fractions
	Nim
Color Tiles	Place Value
	Toolkit
Mirrors	Reflect
	Symmetry
Attribute Blocks	Bridges
	Grids
	Toolkit
Base Ten Blocks	Place Value

Configuration

The configuration option gives the teacher or student control over the use of sound and also the font used in the program. Teachers and students can prepare data disks from the Configuration menu.

Sound Mild sounds are used at various places in the program. The default for sound is for it to be active. The sound status can be switched from the configuration menu. To make the sound inactive choose 'Off' from the Sound menu. To restore sound select 'On'.

Font Three different styles of characters can be used in the program. This option is provided so that

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teachers or students may select the font which is most readable on their particular hardware. The options are Plain, Roman and Bold.

Data_Disk Some of the Hands-On Math activities provide the option to save and load information. The data disk provided with Hands-On Math has sample files but teachers and students will want to create more. Use this menu option to properly format a new data disk for use in saving files. After selecting Data_Disk, choose the drive where the data disk is located and confirm that you want to completely erase the disk. The initialization operation takes a few moments.

Introducing the Playgrounds

Five different Hands-On Math playgrounds are provided in this program. Once Playground has been selected from the main menu five options become available. After the student chooses a manipulative the playground program for that manipulative is automatically loaded. Teachers should decide the best sequence for using the Playground programs based on the needs of the child.

★ Playground: Two Color Counters ★

In the Two Color Counters Playground colored rods are used to represent numbers. When the program is started the options Begin, Options, Exit are presented on the menu bar. After Begin has been selected the menu bar shows three options: Place, Flip and Move. To select a particular option move the indicator to the corresponding choice in the menu and press the return key.

Place   Place first allows the user to select which side of the two color counter is placed face up. The counters are green on one side and violet on another. After either green or violet is selected an indicator appears on the grid. The arrow keys are used to move the indicator and the return key is used to place the counter. Escape is used to cancel.

Flip   Flip allows the user to select a counter that has already been placed and to flip it.

Move   When Move is selected the user can select a counter that has already been placed and move it to a new position.

The second option on the first menu is Options. From this menu several of the special features are activated.

Clear   Select Clear to remove all the counters from the screen.

Flip_All   The Flip_All option flips all the counters that have been placed. It is a good way to show fact families and other mathematical ideas.

Toss   This choice allows the student to select from 1 to 12 two color counters or to let the computer select a random number of counters and toss them on the screen.

Colors   The Colors option lets the user change the counters from green on one side and violet on the other to orange on one side and blue on the other.

Ok   Choose Ok to leave the options menu.

Step-by-step:

1. Select Begin and the choices Place, Flip, and Move appear on the menu bar. Select Place. The computer will give the options Green or Violet so that the user can select which side of the counter to place face up. Choose Green.

2. A flashing indicator appears on the screen. Use the arrow keys to move the indicator to a spot on the grid. Press return to place a counter. Continue moving and placing counters until five counters have been placed.

3. Press the escape key to reactivate the menu bar. Choose Flip and use the arrow keys to move the indicator to one of the green counters. Press return to flip the counter. Flip three counters, then press the escape key .

4. Select Move from the menu bar. Use this option to group the counters in two sets by color. To move a counter get the indicator on the counter and press the space bar, then move to the new position and press return.

5. The counters now show the math fact: $3 + 2 = 5.$

Instructional Applications

The use of concrete materials is one of the best ways to develop a conceptual understanding of mathematics. Two color counters offer a rich source of ideas that can be used with elementary students. Basic number concepts that can be presented with two color counters include matching, grouping, and comparing. Two color counters can be used to introduce students to operations with whole numbers. Effective teaching strategies for presenting more advanced concepts such as fractions, ratios and probability are also possible with the aid of two color counters.

With most math manipulatives a free play period is very beneficial. Free play provides the students with an opportunity to use the computer in a creative way and to become familiar with the control techniques implemented in the software. It would be beneficial for teachers to just observe students during free play time. Much can be learned about individual learning styles and the different approaches to problem solving employed by the students.

During the free play period students will resolve many of the questions they might have about the operation of the program. They will learn how to make selections from the menu, how to clear the screen and how to move and flip counters. Most importantly students will learn how to use the options menu to toss counters and generate problems for themselves. While engaged in free play students will spontaneously place the counters in patterns. This type of activity should be encouraged. They will make discoveries about number combinations that will be important in subsequent activities. They will also discover important relationships pertaining to equivalency and 'greater than' and 'less than' comparisons. Students may begin to investigate the concept of symmetry. Perhaps they will begin to explore important ideas related to probability.

Teachers will find that it is helpful to apply a consistent terminology to discussions of two color counters. Here are some of the terms that are useful.

Color can be used to define a <u>set</u>.
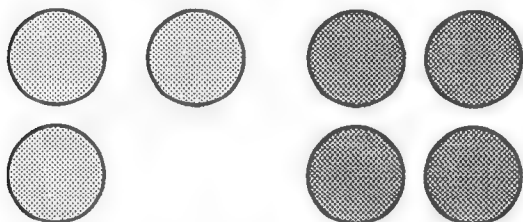
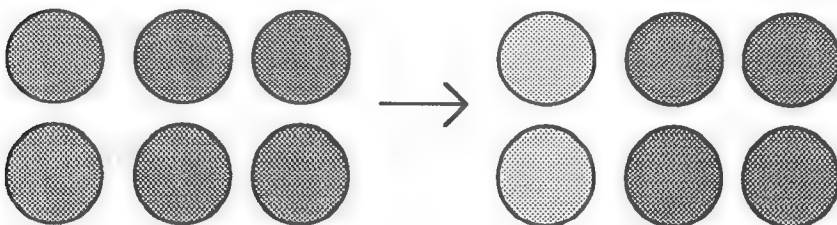Changing colors is called a <u>flip</u>.

Changing position is called a <u>move</u>.
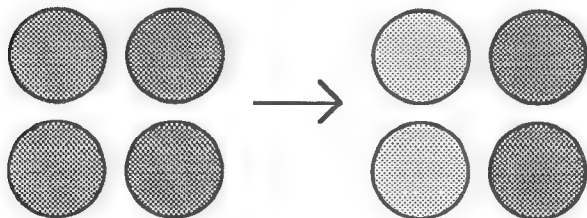
<u>Combining</u> two sets describes an <u>addition fact</u>.

Flipping part of a set describes a <u>subtraction fact</u>.

A set of counters of one color describes a <u>denominator</u>.
Flipping part of the set describes a <u>numerator</u>.

While freely exploring with the counters program students will make many discoveries. Here are some basic concepts:

1. Any set of two color counters represents a number.

$$= 5$$
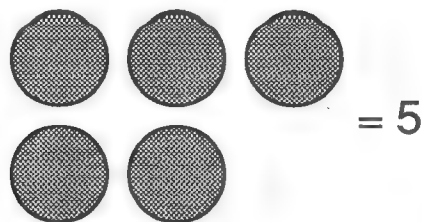
2. For any set of counters, some of the counters can be flipped to describe an addition or subtraction fact.

$$2 + 2 = 4 \quad \text{or} \quad 4 - 2 = 2$$
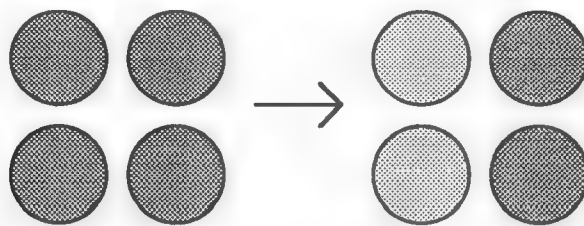
3. When some of the counters in a set are flipped, each of the resulting addition and subtraction facts can be thought of two ways. (Fact Families)

$$2 + 3 = 5 \quad \text{or} \quad 3 + 2 = 5$$
$$5 - 3 = 2 \quad \text{or} \quad 5 - 2 = 3$$
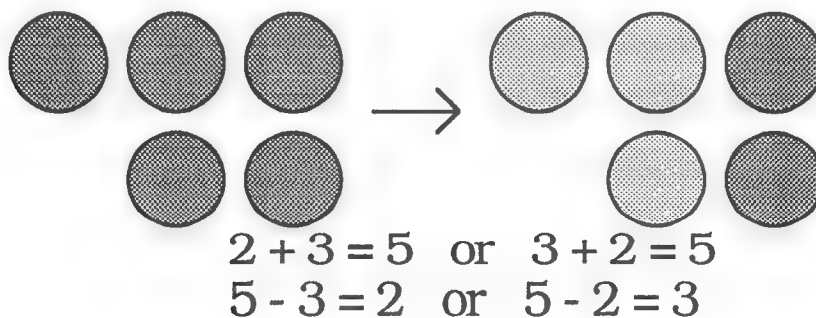
4.   Flipping some of the counters in a set represents a fraction.

$$\frac{3}{5}$$

The reproducible activity pages suggest ways for teachers to provide guidance as students explore the many mathematical concepts that can be studied using two color counters.

☆ Playground: Color Tiles ☆

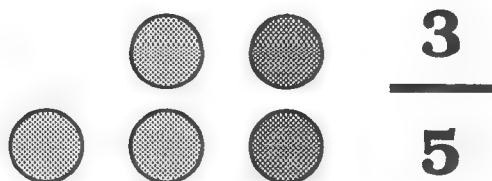When the Color Tiles Playground is selected from the main menu the screen will show the menu for the playground which consists of Begin, Options, and Exit. The Color Tiles Playground consists of a 12 x 8 grid where color tiles can be placed. The tiles are 6 different colors: green, violet, orange, blue, black and white. After Begin is selected the menu bar will show the names of each color. Select a color and then move to the desired position and press return. To change colors press escape, select the new color, move to the desired position and press return.
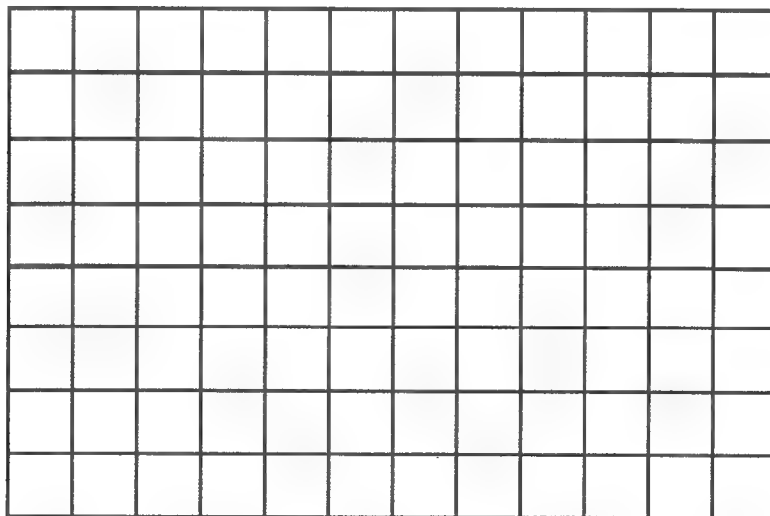
Color tiles are a very useful math manipulative that can be used to develop many skills including: counting, addition, subtraction, place value, multiplication, division, fractions, patterns, area, probability, and graphs. In addition to allowing the student to select a color tile and place it anywhere on the grid, the program provides some interesting options.

Preferences and special functions are available from the Options Menu.

Clear   Select Clear to remove all the tiles from the screen and reset the memory.

Automatic_Draw Clears the screen of all tiles and sets up the computer to automatically draw a design. Each time the space bar is pressed a randomly selected color tile is placed at a randomly selected position on the screen. The escape key is used to exit from the Automatic_Draw mode. The symmetry setting is used in the Automatic_Draw mode.

Symmetry   When symmetry is selected the menu bar will show four options: Horizontal, Vertical, Both and None. Each option describes the type of symmetry that is used in making a design. The symmetry setting affects the manual and automatic draw modes.

Horizontal   Vertical   Both
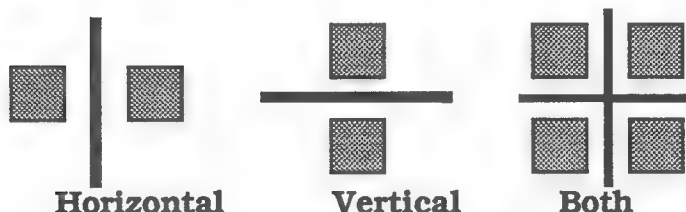
Shift Shift automatically changes the color of each tile that has been placed. The pattern remains the same but the colors are all shifted.

Step-by-step:

1. Choose Symmetry from the options menu and select Both. Choose OK and then select Begin from the menu bar. Now the menu bar shows the 6 choices of color tiles. Select green and a flashing indicator will appear in the first cell of the grid.

2. Use the arrow keys to move to the center. Make a group of four green tiles in the center of the grid by moving to the appropriate location and pressing return to place a tile.
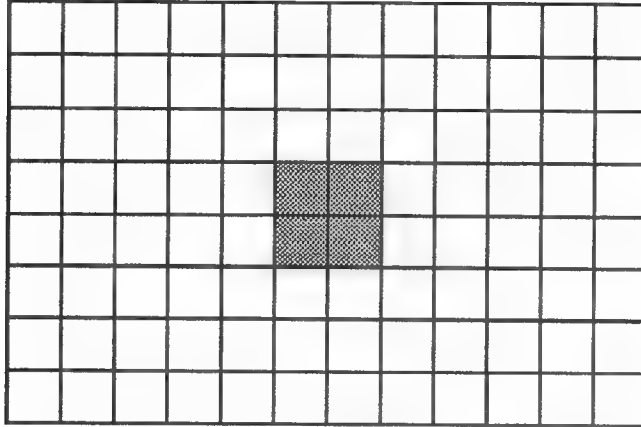
3. Press the escape key to reactivate the menu bar. Choose violet color tiles and surround the green square with twelve violet color tiles.
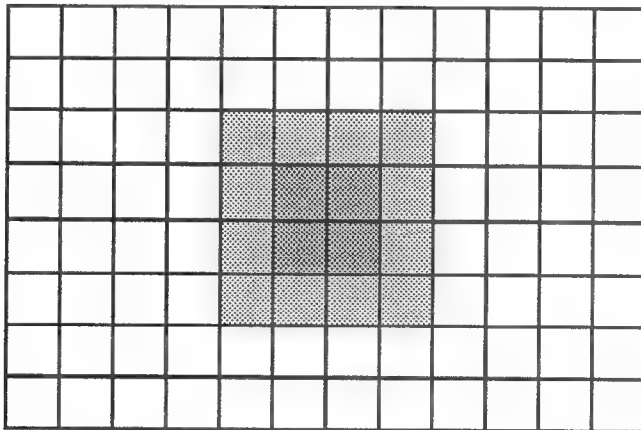
4. Use the escape key to get back to the menu and choose blue. Complete the design adding eight blue tiles as shown.

5. Press escape twice and choose Options from the menu. Select shift to change all the colors in your design. Choose Clear to remove all the tiles.

Technical Note: *The standard Apple // video display uses two sets of high resolution colors: group 1 (green, violet, black1 and white1) and group 2 (orange, blue, black2, and white2). Bleeding (a distortion on the screen) occurs when a color from group 1 is placed next to a color from group 2.*

Instructional Applications

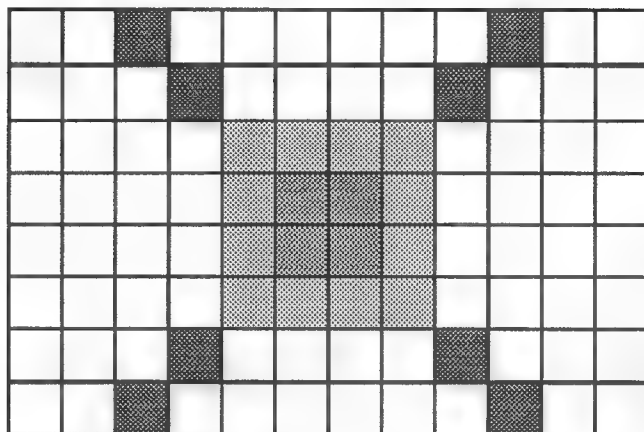Effective mathematics education begins with active involvement with concrete objects. Through the use of manipulative materials children develop an intuitive understanding of mathematical concepts. Color tiles are an interesting tool for exploring a variety of concepts. Teaching strategies that include the use of color tiles will prove to be rewarding for teachers and students. Children will benefit from extended free play.

If teachers observe students while they are engaged in free play with the color tiles they will notice that children spontaneously discover important mathematical ideas. Here are a few of the possibilities.

1. A tile can be placed to the left of other tiles or to the right of other tiles. Tiles can be over or under other tiles. Tiles can be above or below other tiles. Tiles can be between other tiles. Students can learn the meaning of first, second, and third place as well as beginning or last.
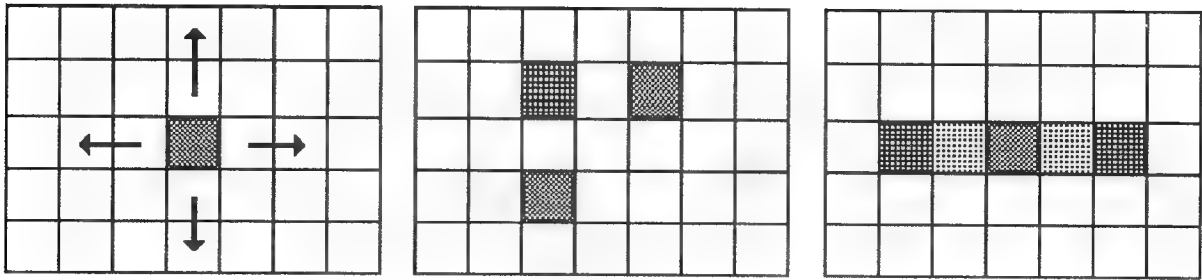
2. Two groups can have the same number of objects. (Equivalence) Some sets have more objects than other sets. (Greater than) Some sets have less objects than other sets. (Less than)
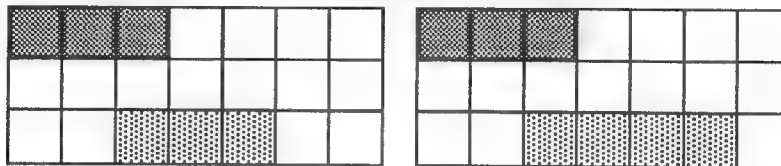
3. Number symbols can be used to tell how many objects are in a set.

= 6
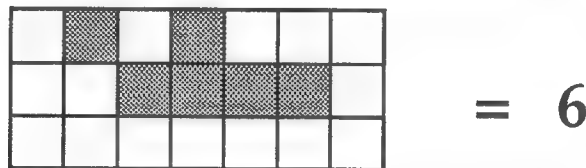
4. Some designs are *symmetrical* and some designs are *asymmetrical*.
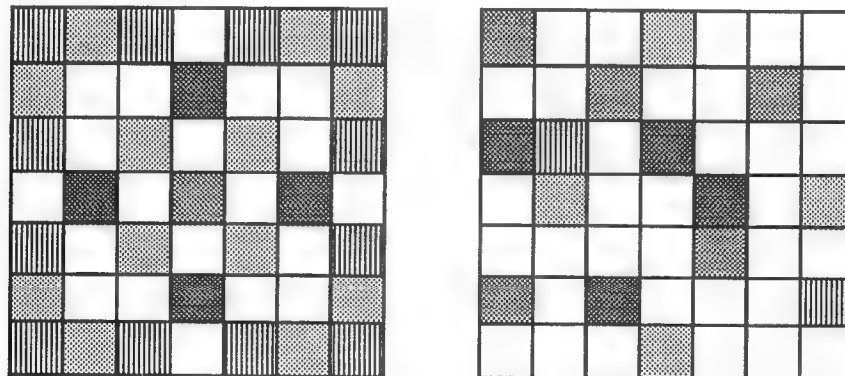
5. Addition facts can be shown with color tiles.

___ + 6 = 8

2 + 6 = 8
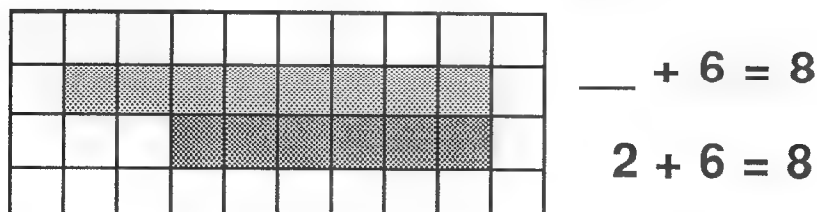
6. Color tiles can be used to represent place value. These tiles represent 356.

$$1 \ \blacksquare = 10 \ \square$$

$$1 \ \square = 10 \ \blacksquare$$
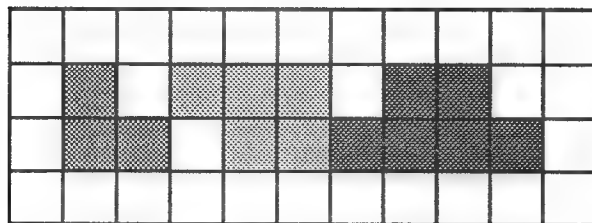
7. Rectangular arrangements of tiles can be used to show multiplication.
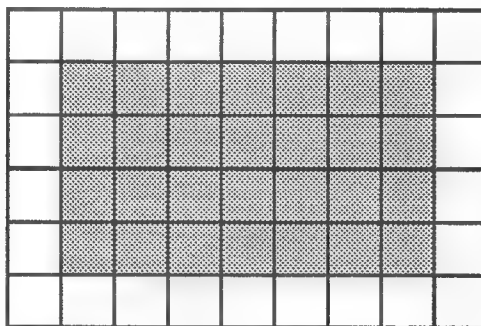
4 x 7 = 28

8. Arranging a set of color tiles in equal groups can be used to show division.
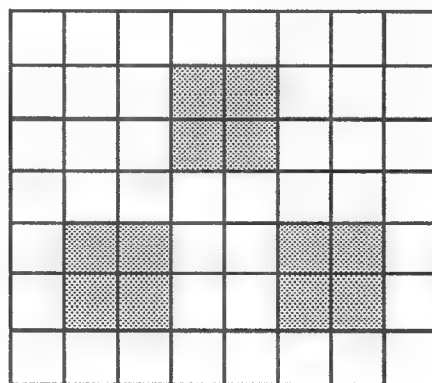
12 ÷ 3 = 4

★ Playground: Mirrors ★

When Playground and then Mirrors are chosen from the menu bar the Mirrors Playground is executed. The program creates a large open area on the screen with a vertical line indicating the position of the mirror. Choose Begin to start a new drawing. These keys are used when drawing a figure:

Arrow Keys	*Move the drawing point up, down, left and right.*
Return	*Marks a vertex in the figure by anchoring the drawing point to either begin or end a line segment.*
Delete	*Removes the last line segment. Delete is only active when the mode is set to Manual.*
Escape	*Escape exits the draw mode and restores the menu. If the mode is set to Manual then options Reflect or Cancel are given.*

With the mirror set to vertical, objects drawn on one side of the mirror are reflected onto the opposite side.
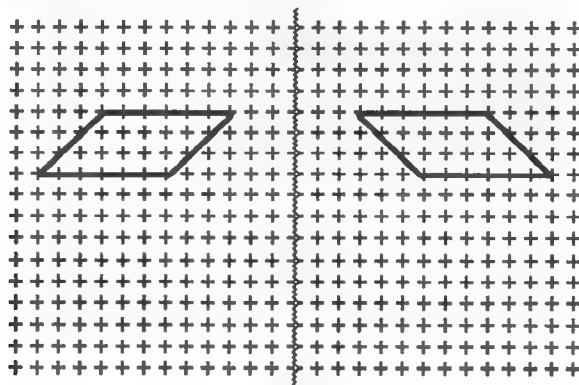
Preferences and special functions are available from the Options Menu.

Clear   Select Clear to remove all the objects from the screen and reset the memory.

Mirror   The settings for mirror are: Vertical, Horizontal, Both or None.

Vertical   *Horizontal*

Both   *None*
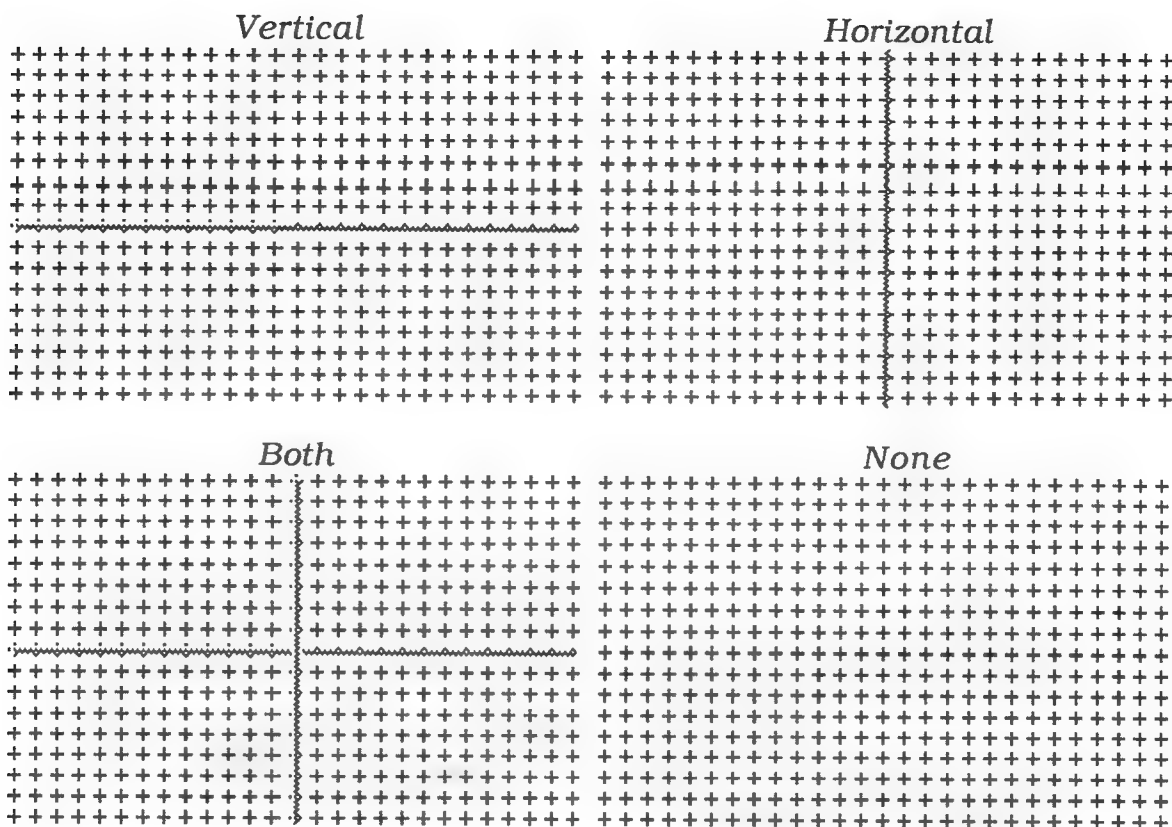
Mode   The mode setting controls whether the drawing is automatically reflected as the object is drawn or whether it is manually reflected after it is drawn.

Color   The color of the drawing pen can be set to green, violet, orange, blue or white.

Step-by-step:

1. Choose Options from the first menu and then set the pen color to green. Leave the mirror and mode settings at their default setting, which is vertical and automatic. Select Ok and then choose Begin.

2. Use the arrow keys to move up two and to the right two. Press return to start drawing the parallelogram shown below.
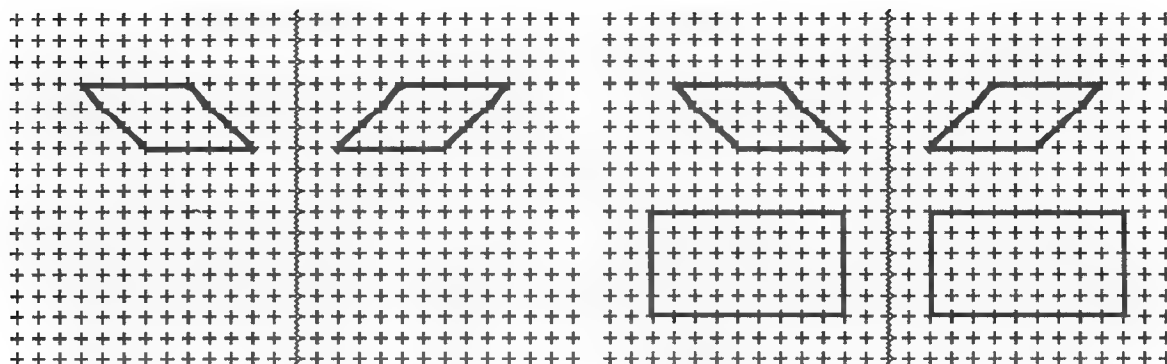
3.   Move to each vertex (corner) of the parallelogram and press return. The drawing is automatically reflected in the vertical mirror.

4.   To draw the rectangle that is shown below the parallelogram, first press the space bar to lift the pen. While the pen is in the pen up mode move to the starting place for the new figure and press return to start drawing.

5.   Again move to each of the vertices in the rectangle and press return. The drawing is automatically reflected to the left side of the grid.

Instructional Applications

When children freely explore with the Mirrors Playground they begin to develop an intuitive understanding of important geometrical concepts. Many of the postulates used to develop proofs in formal geometry rely on the reflexive and symmetric properties of equivalence and congruence. Using the Mirrors Playground, students can develop visual thinking skills by making a drawing of a geometric figure and predicting the reflection of the figure.

Much of the difficulty that students have in geometry stems from a lack of skill in visually interpreting geometric diagrams. The Mirrors Playground helps to develop a fundamental understanding of geometry because students define figures by determining the vertices and line segments that comprise the figure.

Teachers can use the Mirrors Playground to help students master a variety of geometry objectives. Here are a few ideas of ways to use the program:

1. Learn the names of geometric figures by drawing and reflecting a square, rectangle, right triangle, isosceles triangle, parallelogram and trapezoid.
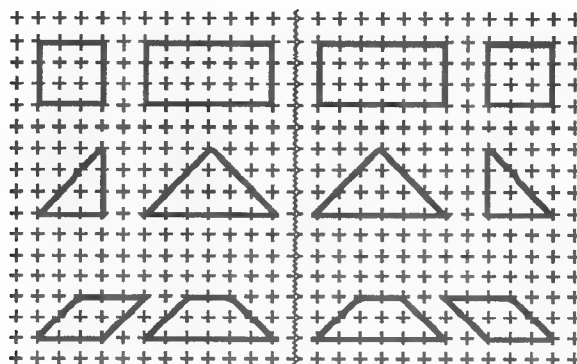
2. Develop visual thinking skills by learning to predict the reflections of a figure.
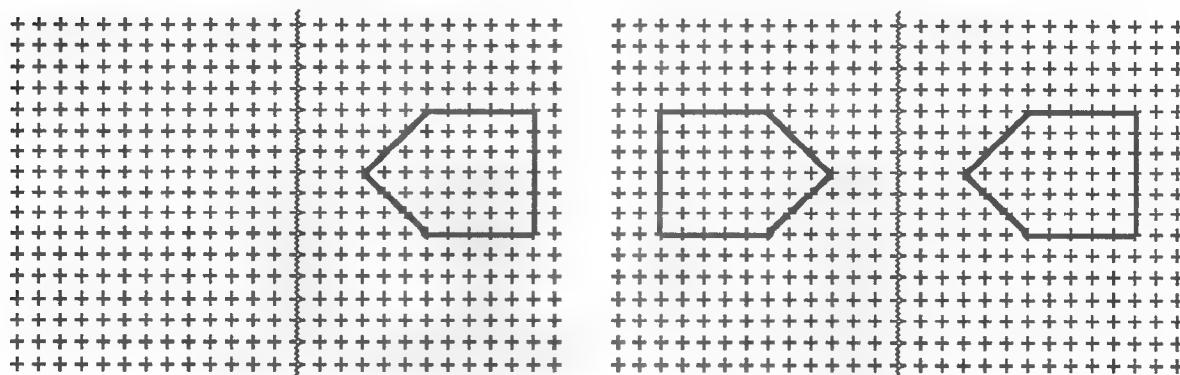
3. Learn to recognize bilateral symmetry.
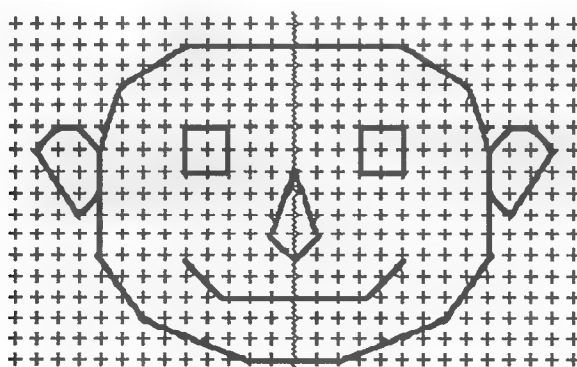
4. Tell whether a geometric figure is symmetrical or not.
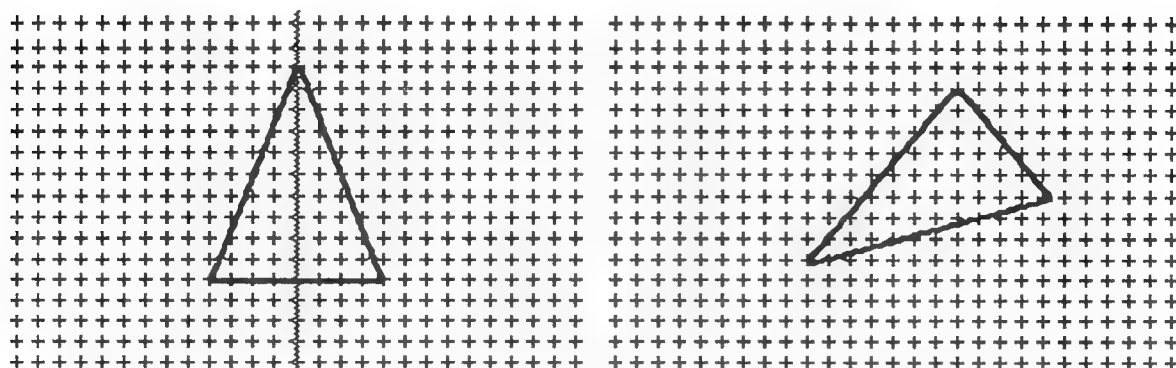
★ **Playground: Attribute Blocks** ★

The Attribute Blocks Playground helps to build logical thinking skills. The set of attribute blocks used in the program has 32 pieces. The complete set consists of four colors, four shapes and two sizes. Pieces are selected by selecting the attributes of the piece from the menu bar.

‖‖
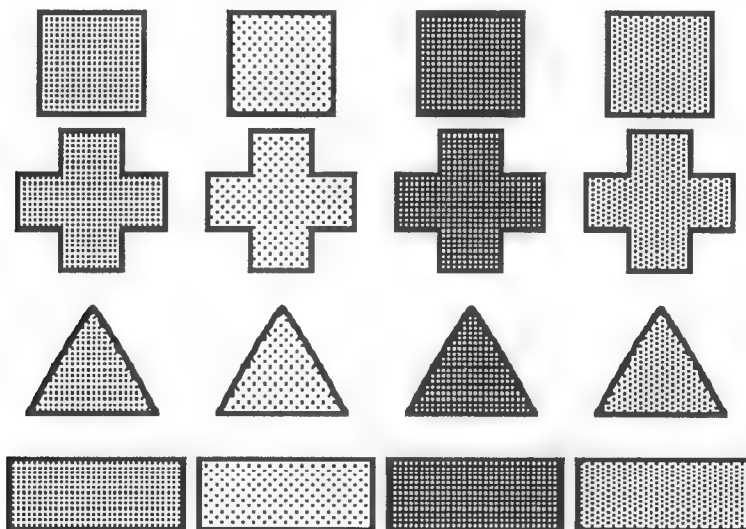
The Options menu for the Attribute Blocks playground enhances the educational value of the program.

Clear   Removes all the attribute blocks from the playground and resets the trail and difference settings.

Trail   After Trail is selected the letters A through J appear on the menu bar. Each letter represents a different trail. When a trail is selected the object of the playground is to place pieces along the Trail in accordance with the difference setting.

Difference The Difference setting determines how many logical differences there should be in the attributes of each piece that is placed along a trail. It can be set to 1, 2 or 3. The Difference setting is used only in conjunction with a trail. The default setting is one difference.

Limit   The Limit setting can be either On or Off. When the setting is On each attribute block can be used only once. When the setting is Off pieces can be used repeatedly. Teachers can use this option to control the difficulty of the activity.

The first step-by-step example will demonstrate how the pieces can be placed on the Attribute Blocks Playground in a pattern based on shape. Similar patterns can be placed on the playground using color or size.

Step-by-step:

1. The first step is to place a large green square on the playground. Select Begin. Select Large and then Green. The indicator highlights a large green square. Press return and a flashing square appears on the playground.

2. Use the arrow keys to move the square down one row. Press return to place the square.

3.   Choose Large, Green and Cross. Place the large, green cross to the right of the large, green square.

4.   Now choose Large, Violet and Square. Place the large, violet square to the right of the large, green cross. Which piece comes next in the sequence?

The second Step-by-step example uses a trail. Pieces are placed along the trail according to the difference setting.

Step-by-step:

1.   From the first menu select Options and Clear to erase the playground and reset the program.

2.   Choose Trail and select A, the first trail. A path appears on the screen as shown below. The green light is the starting point and the stop sign is the goal.

3.   Set the Difference level to 1 by choosing Difference and then 1. Select Ok to exit the options menu.

4. Choose Begin and select Large and Green and then Square. Place it on the green light. The first piece must always be placed on the green light.
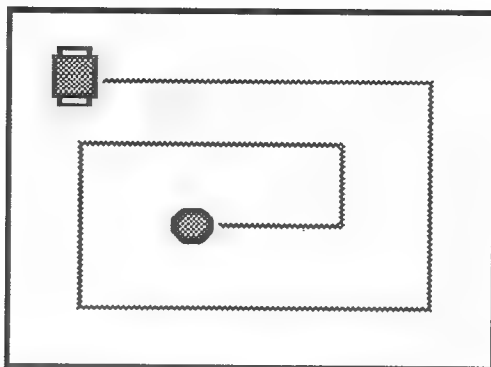
5. Now choose a piece that has one attribute that is different from the large, green square. A large, green cross has one different attribute - shape. Place a large, green cross on the path next to the large green square. Place a large, violet cross next to the large, green cross.
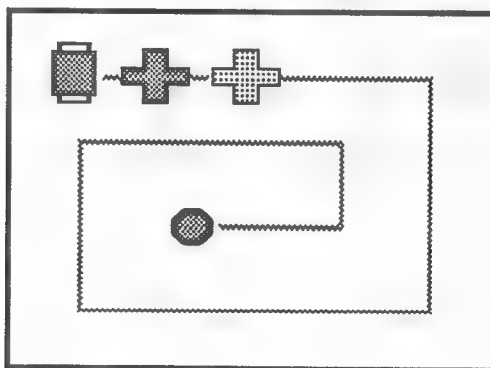
6. Continue placing pieces along the path until you reach the stop sign.

Instructional Applications

Logical thinking ability is a skill that is extremely important for children to develop. Using the Attributes Blocks Playground children can develop math readiness skills. They can learn to classify and sort objects by size, color and shape. They can learn to repeat patterns based on size, shape or color. Young children can learn to recognize and name the basic shapes. Teachers can use the program as an activity where children practice counting skills. When using the Trails option children are challenged to place attributes blocks along a

path by applying a logical rule. By playing this game children develop an understanding of critical differences.

Here are a few ideas of the kinds of activities in which teachers can engage their students using the Attribute Blocks Playground:
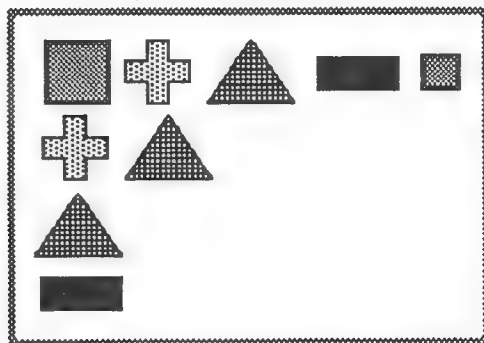
1. Children can experiment with logical patterns by placing pieces on the screen in rows and columns.
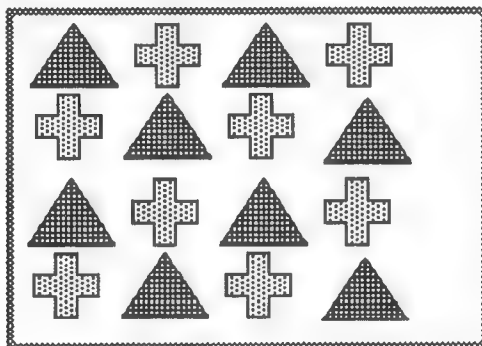
2. Children can place pieces in a row to show a logical sequence. Here each piece is different in two ways.
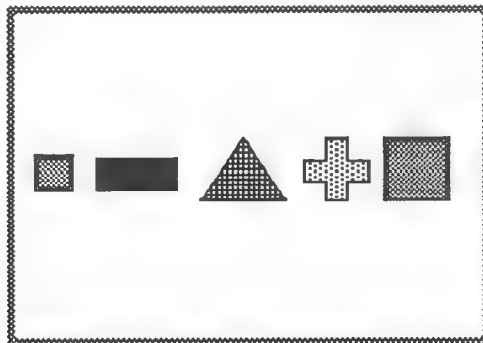
3. Children can place pieces along a trail using a 1, 2 or 3 difference rule.
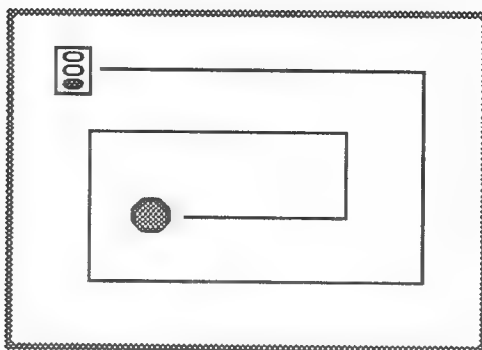
★ Playground: Base Ten Blocks ★

The Base Ten Blocks Playground helps the child develop an understanding of place value. In order for a child to develop a meaningful understanding of mathematics it is essential that the child know the underlying concepts that are the cornerstone of the representational place value system. After a child has developed a clear understanding of addition and subtraction as operations involving the joining and separating of sets, he is ready to begin the systematic study of numbers greater than 9.

The decimal system employs only ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Children must learn that the position of a given digit in a number determines its value. For example, in the number 387 the 3 represents 3 sets of one hundred, the 8 represents 8 tens and the 7 represents 7 ones.

The Base Ten Blocks Playground provides children with an opportunity to freely explore place value concepts. The Playground uses four types of blocks.

| Thousands | Hundreds | Tens | Ones |

‖‖
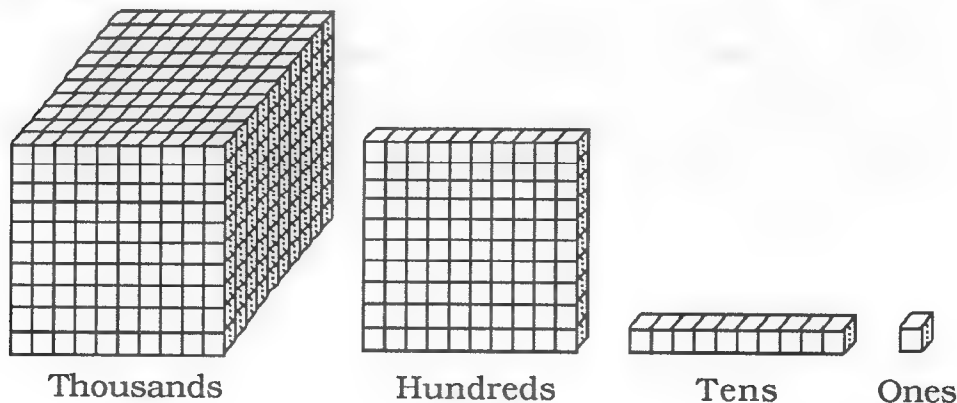
Using this special playground the child can select and individually place base ten blocks anywhere on the screen. After Begin is selected from the menu bar the options Thousand, Hundred, Ten and One appear. If Thousand is selected the computer will display a thousands block. Using the arrow keys the block can be moved up, down, right and left. Return places the block and restores the menu. In a similar way hundreds, tens and ones blocks can also be placed.

The Options menu provides access to these operations:

Clear   Select Clear to remove all the blocks from the screen and reset the memory.

Count   Count the blocks that have been placed to determine the value of what is showing on the screen.

Arrange   The arrange option allows the student to select and move blocks that have already been placed.

This example will demonstrate how the base ten blocks can be placed on the Base Ten Block playground to represent a specific number. Teachers will want to lead children in similar activities which can be very helpful in developing a fundamental understanding of place value. In this example you will use the Base Ten Blocks to represent the number 2,375.

Step-by-step:

1. After selecting the Base Ten Blocks Playground from the menu the menu bar will display the choices Begin, Options, and Exit. Choose Begin.

2. The next menu offers the choices: Thousand, Hundred, Ten and One. Since the number that we want to represent has 2 thousands, press return with the thousand choice highlighted. A thousands cube appears on the screen. Use the arrow keys to move the block up and press return to place the block. (Note: To avoid confusion it is best to always move a block away from the starting position.)

Thousand Hundred Ten One
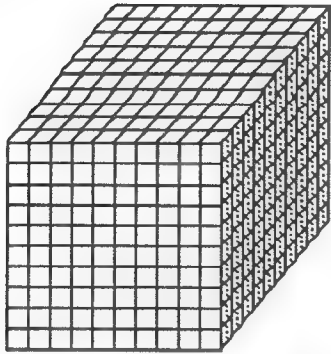
3. Continue selecting blocks from the menu bar and placing them on the screen until 2 thousand blocks, 3 hundred blocks, 7 ten blocks and 5 one blocks have been placed.

Thousand Hundred Ten One
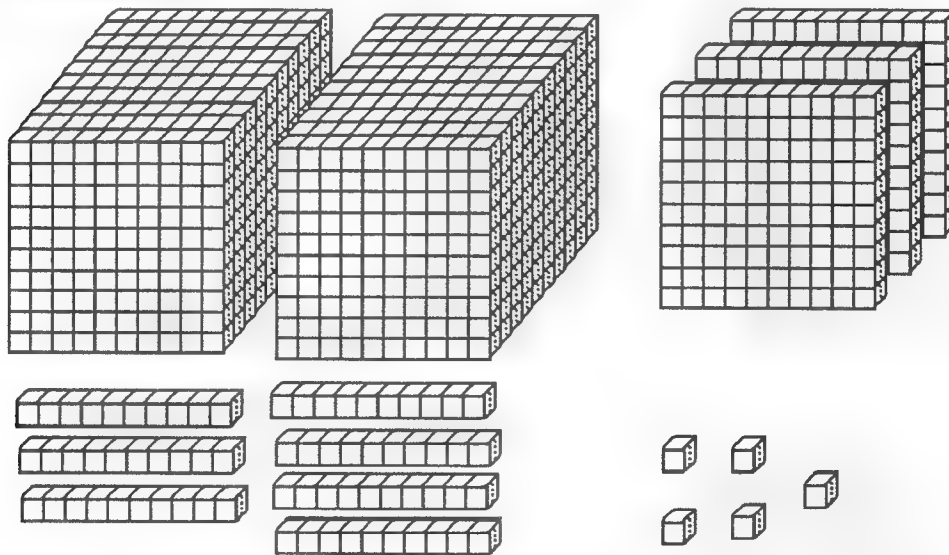
4. After all the blocks have been placed press the escape key and the menu bar shows the choices: Begin, Options, Exit. Choose Options and then Count. Watch as the program steps through each block that has been placed and adds the value of the block to the counter. The number 2,375 will be shown on the menu bar. Press any key to restore the options menu. Choose Clear and then Ok to try another problem.

Instructional Applications

The place value system is based on the concept of groups. Using Base Ten Blocks, children develop an understanding of grouping in powers of ten. Activities which involve representing numbers using Base Ten Blocks or telling which number is represented by a set of blocks reinforce a child's comprehension of the decimal number system.

Because the computer graphics used in the program allow objects to 'pass through' other objects children can easily discover that a ten block may be exchanged for 10 one blocks. By manipulating physical materials and then by simulating the manipulation of physical materials using the computer children are given the opportunity to internalize the basic ideas of the decimal place value system.

Some suggestions for worthwhile activities are the following:

1. Tell the name and give the dimensions of each block.
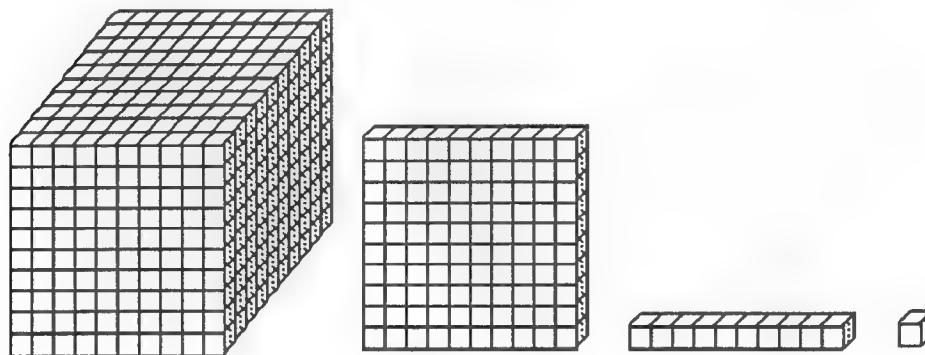
2. Give the value of the number represented by a set of blocks.

$$= 457$$
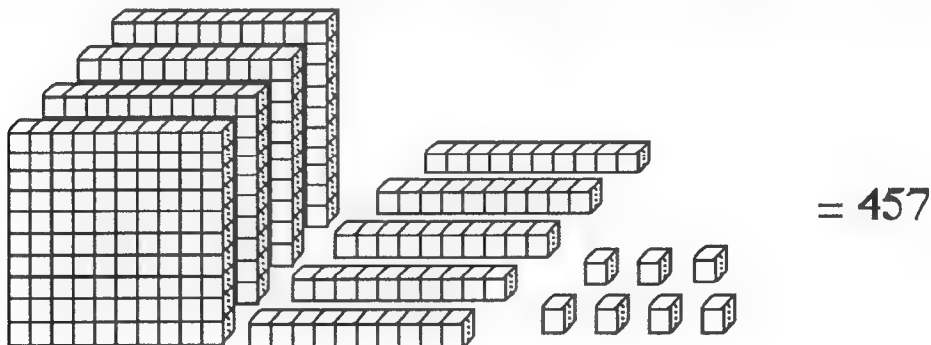
3. Represent a given number with blocks.

$$345 =$$
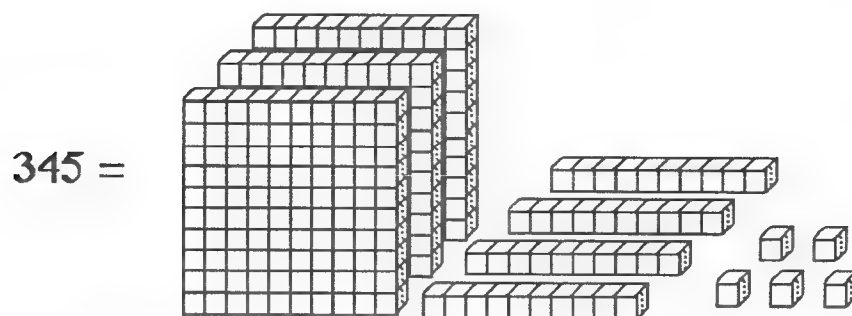
Introducing the Exercises

Hands-On Math: Volume II includes several exercises involving the five manipulative devices used in the program. Once Exercise has been selected from the main menu the five options for manipulative devices appear.

Two Color Counters	Skills Fractions Nim	Color Tiles	Place Value Toolkit
Mirrors	Reflect Symmetry	Attribute Blocks	Bridges Grids Toolkit
Base Ten Blocks	Place Value		

Two Color Counters: Skills

Objectives: Addition skills: one digit numbers with and without regrouping. Define an addend and a sum. Quick recall of addition facts. Subtraction skills: Differences with numbers up to 18. Define a subtrahend and minuend. Quick recall of subtraction facts.

The Two Color Counters Skills Exercise is an addition and subtraction drill program that uses two color counters to represent addition and subtraction problems. Special options in the program allow the teacher to control the range of the random numbers used in the problems generated by the program. The option allows teachers to customize the program so that an individual child's activity is directly aimed at appropriate developmental skills.

The Options menu offers these choices:

Clear   Select Clear to remove all the Two Color Counters from the screen. The range and skill selections are not changed.

Range   The range option gives the teacher or student control of the range of numbers used in the program. After Range is selected the next choice is First Addend or Second Addend. (Subtrahend or Remainder for subtraction) Once a digit is selected the next choice is Minimum or Maximum and finally the set of numbers 0-9 appears on the menu bar.

To set up the program to drill the student on addition combinations where the first addend is five, follow this procedure:

Select Options and then Range. Select First Digit, then Minimum. When the list of numbers appears select 5. Again select Range, First Digit and then Maximum. Select 5 as the maximum. Set the range for the second digit to 0-9. Now whenever problems are selected the first digit will be a five and the second digit will be a randomly selected number in the range 0-9.

If the user attempts to set the program for an illogical range, for example, where the minimum is higher than the maximum, the program resets itself to the default range which is 0-9 for each digit.

Goal   The number of problems required to complete an exercise can be set using the Goal option. The goal can be 5, 10, 15, 20, or 25 problems.

Skill   The skill practiced in the drill can be switched from addition to subtraction. When the skill is set to addition the range setting applies to the first and second addend. When the skill is subtraction the range setting applies to the subtrahend and remainder.

Addition Skills

When using the Two Color Counters Skills program to develop addition skill the Option menu should be used to set the skill to addition and the desired range and goal should also be set. Select

Begin from the first menu to start the drill. A screen similar to the one shown below will be displayed.

Enter an addend.
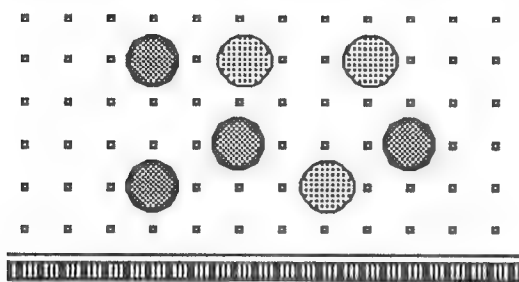
Enter an addend.
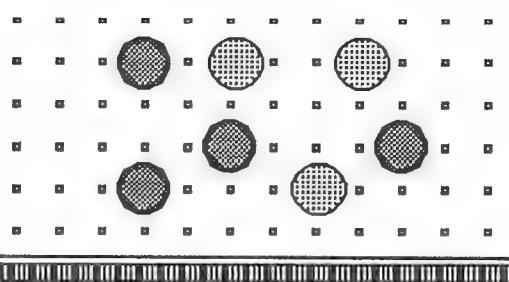
? + □ = □   3 + ? = □

Enter a number to tell how many of one color of counter is displayed. The number can be entered by typing it on the keyboard or by using the right and left arrow keys to increase or decrease the number. Press the return key to complete the entry. If the number entered corresponds to one set of counters it is accepted and the question mark cursor moves to the next entry. If the number entered is incorrect the computer will beep and wait for the correct answer to be entered.

After the two addends have been correctly entered the cursor moves to the answer position and allows the student to enter the answer. Press enter after each digit when the answer is a two digit number.

Subtraction Skills

When the skill is set to subtraction the program first displays a set of two color counters with the green side up. The direction "Enter the number of counters. (Minuend)" appears at the top of the screen. After the student correctly enters the number of green counters, a certain number of the counters are flipped. The student is directed to enter the number of counters that have been flipped. After the subtrahend is correctly entered the student is asked to enter the remaining green counters.

||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||
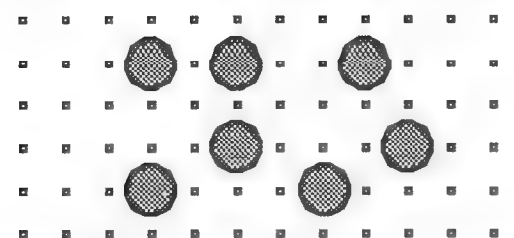
Enter the number of counters. (Minuend)   **How many are flipped? (Subtrahend)**
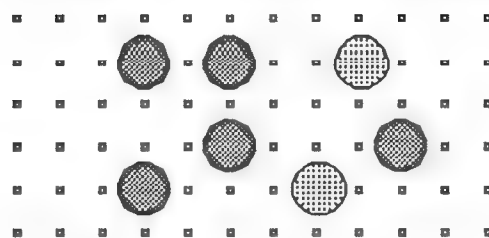
$? - \square = \square$ $7 - ? = \square$

When the goal is reached in either the addition or subtraction drill a congratulatory message appears and the program returns to the first menu.

Teachers will want to use the Options menu to tailor the Two Color Counters: Skills program to better meet individual needs. The range option can be used to control the range of numbers used in the problems. When children are first learning addition or subtraction many teachers will want to limit the range of numbers to 1 through 5. Using zeros in problems can be confusing to young learners. Teachers will want to discuss problems with zeros before allowing them to come up in the program. To avoid zeros use Options, Range, Minimum to set the minimum to 1.

Two Color Counters Fractions

Objectives: Practice defining a fraction as a rational number. Define and demonstrate an understanding of a fraction as an expression for a part of a whole. Learn the names of common fractions. Simplify fractions by selecting the greatest common factor (GCF).

The Two Color Counters Fractions Exercise displays a set of counters on the screen, flips a part of them and asks the student to give the fraction that describes how many counters have been flipped.

The Options menu offers these choices:

Clear   Select Clear to remove all the Two Color Counters from the screen. Other settings are not affected by clearing the screen.

Goal   The Goal setting controls how many problems are needed to complete the exercise.

Zero   The Zero option controls whether a numerator can be zero. When the Zero option is set to 'On' zeros are possible. When the option is 'Off' zeros are not used. Problems with a numerator of zero may be confusing to some children and require teacher explanation.

Maximum   The Maximum option controls the size of the denominator in the fractions which are used in the drill.

Reducing   When Reducing is set to 'On', the menu bar offers the options Reduce and Ok after the student enters the denominator and the numerator. If the student chooses Reduce, the menu bar shows a list of numbers. The student can then pick a number and the fraction is reduced by that amount if the number is appropriate.

When using the Two Color Counters Fraction exercise to develop an understanding of the meaning of a fraction, the teacher will want to use the Options menu to individualize the student's experience with the program. The drill first involves the student in giving the denominator by answering the question, 'How many counters in all?' Next, some of the counters are flipped and the student must enter the numerator. Finally if the Reduce option is set to 'On' the student is asked if the fraction should be reduced or not. If reduce is selected a list of numbers (GCF's) are presented on the menu bar.

IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII

How many counters in all? (Denominator)   **What part has been flipped? (Numerator)**
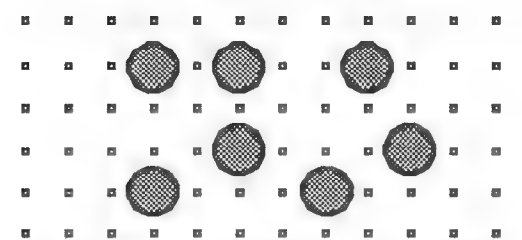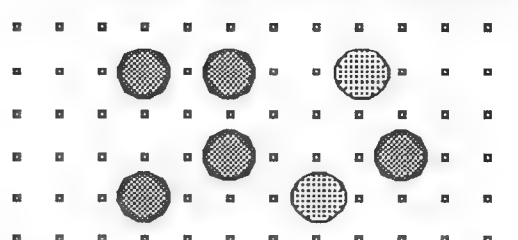
$$\frac{\square}{?}$$ $$\frac{?}{7}$$

Two Color Counters Nim

Objective: Win a strategy game when playing against the computer.

One of the oldest strategy games known to man is a game called Nim. The game is believed to have originated in ancient China. The first mathematician who analyzed the game gave it its name which is from an archaic English verb which means to steal or take away. In this adaptation of the game the two color counters are used.

The Gameboard and Rules

The Nim gameboard consists of three rows of counters.   Three counters are placed in the first row, five in the second and seven in the third row. As counters are selected they are flipped and cannot be selected again during that game.

Begin Options Exit
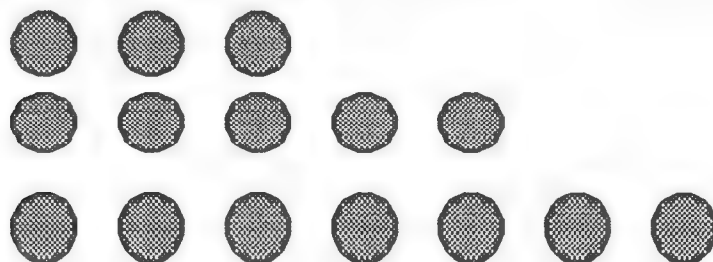
The rules for the game are simple. A player may flip as many counters as he wishes but only from a single row during a turn. Players may not flip counters from two different rows during the same turn.

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To start the game select Begin and a flashing indicator appears on the first counter. Using the arrow keys the player may move vertically to a different row or horizontally to select counters. To flip a counter press the return key. Once a counter in a particular row has been flipped, the player can no longer move vertically during that turn. Any number of the available counters in the same row can be flipped. The spacebar is used to pass the turn. The computer 'thinks' for a moment and then makes its move. Play continues until the last counter has been flipped.

A variety of options are provided to enable the student to change the way the game is played:

Clear   Select Clear to remove all the Two Color Counters from the screen. Other settings are not affected by clearing the screen.

First_Player   This option allows the player to decide who goes first. The choices are Human or Computer.

Win_Option   The Win Option controls the object of the game. The choices are 'Last flip wins' or 'Last flip loses'.

Colors   Just for fun two types of two color counters can be used. One set is green on one side and violet on the other. The other set is orange and blue.

Children will enjoy trying to figure out how to beat the computer in this game. The winning strategy can be defined mathematically. A good suggestion is for the students to play the game several times and to make a list of the number of counters left after each round in binary notation. A pattern will emerge.

Color Tiles Place Value

Objectives: Develop an understanding of the place value system. To practice counting in multiples of one thousand, one hundred or ten. Recognize the base ten place values.

The Color Tiles Place Value exercise uses a different color of tile to represent a place value. The base for the number system can be changed. The meaning of each color for each number system is described below:

	Base			
	2	5	10	12
Orange	8's	125's	1000's	1728's
Blue	4's	25's	100's	144's
Violet	2's	5's	10's	12's
Green	1's	1's	1's	1's

The program randomly selects a number and represents it as a series of color tiles on the grid at the top of the screen.

Begin Options Exit

1,000's	100's	10's	1's
0	0	0	0

The options available in the program allow the display and goal to be changed.

Clear   The Clear option erases the color tiles from the screen and resets the program.

Display   The Display option allows the labels to be changed from the place value (1,000's, 100's, etc.) to the corresponding color words (Red, Blue, Violet and Green).

Goal   The Goal option allows the user to set the number of problems required to complete an exercise.

|||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||

Base   The Base for the number system used can be
                            changed to give children experience in working
                            with different bases.   The choices are base 2,
                            base 5, base 10 and base 12.   The default base is
                            base 10.

This drill program helps children understand the meaning of the
place value system.   After Begin is selected a number is represented
using color tiles and a flashing indicator appears in the 1,000's
column.   The indicator can be moved to the right or left using the
arrow keys.   If a number key is pressed while the indicator is in a
particular column, the number typed replaces the number that holds
the place.   Using the arrows to move and the number keys to enter
digits the student shows a number equal to the number represented
with color tiles.

When the student is satisfied that the number represented using digits
is the same as the number represented using color tiles he should
press return. The menu bar shows the choices: Check, Edit or Cancel.

If Check is selected the program will check the student's answer and
report any discrepancies and return to the edit mode.   If Edit is
chosen the student's response is not scored and the program returns
to the edit mode. If Cancel is selected the quiz stops and the program
returns to the first menu.

Color Tiles Toolkit

Objectives: Develop an understanding of bilateral symmetry by using
             color tiles to reflect a design. Repeat a pattern based on a
             given color sequence.   Find combinations that equal a
             given number.   Practice addition facts with two or three
             combinations.   Write multiplication facts using color tiles.
             Represent fractions with color tiles. Find the area of a
             figure with color tiles.   State the correct number when
             color tiles are used to represent place values.

The Color Tiles Toolkit allows teachers and students to load and save
screens where color tiles have been placed.   The Options menu
provides the ability to load and save screens as well as a variety of
other choices.

Clear   The Clear option removes all the color tiles from
                            the grid.

Load   The Load option is used to restore a screen that
                            has been stored on the data disk. The Hands-On

Math Data Disk contains 10 screens that are used with the activity pages.

Save
The Save option allows teachers and students to create and store screens. Teachers can create new color tile problems and students can save their work using this option.

Symmetry
The Symmetry option controls the symmetry feature. The options for symmetry are Horizontal, Vertical, Both or None. When symmetry is active color tiles are automatically placed according to the symmetry rules.

Shift
The Shift option automatically changes the color of each color tile that has been placed.

Using the program problems can be set up to develop a variety of mathematical skills. For example, here is a screen where the right half of a design has been created with color tiles. The screen can be stored on a disk and retrieved by a student who has been given the task to complete the left half of the design.
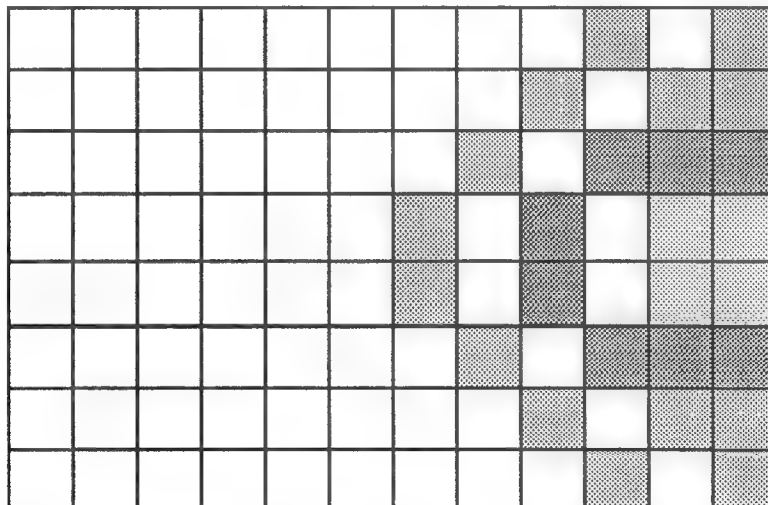
Many learning opportunities can be structured around the use of color tiles. Here for example, the student can be asked to continue this pattern.

Sets of color tiles can be used to represent addition facts. For example, students can be asked to cover an area made from a set of 8

white color tiles with two different colors, thus showing an addition combination.

$$5 + 3 = 8$$
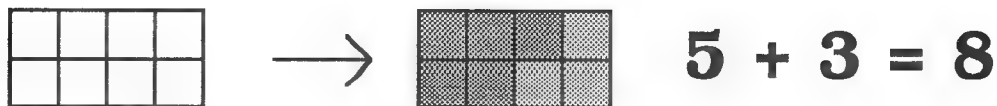
Students can show multiplication facts with color tiles.

$$4 \times 4 = 16$$
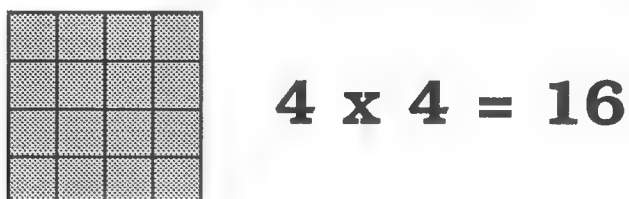
Students can learn to find the area of rectangles using color tiles.

$$A = lw$$

$$24 = 6 \times 4$$
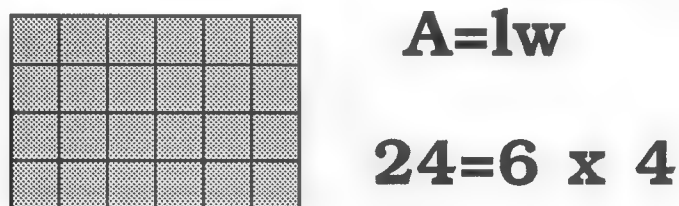
Fraction concepts can be learned with color tiles.

$$\frac{2}{5} \text{ of } 20 = 8$$
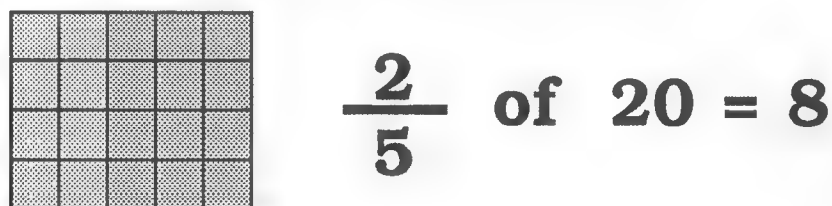
Each color tile can stand for a different place value. A group of color tiles can represent a number using the decimal place value system.

1,000's   10's

100's   1's
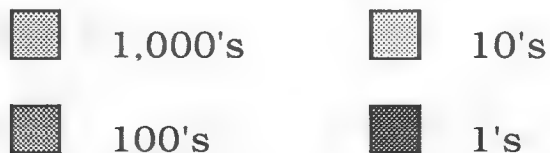
Mirrors Reflect

Objectives: Define and recognize a particular geometric transformation: the reflection. Recognize congruent figures. Define and recognize bilateral symmetry. Define, name and draw line segments, angles, planes, vertices, polygons, horizontal and vertical lines, perpendicular lines, intersecting lines, squares, rectangles, parallelograms, trapezoids and triangles.

The Mirrors Reflect exercise allows a teacher or student to create reflection problems. Once a problem has been created it can be solved and checked by the computer. An example of a reflection problem is shown below where the object is to draw the reflection of the figure shown on the right.

Draw Mode
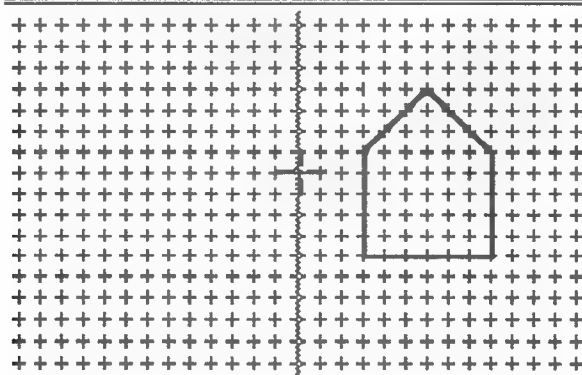
Draw Mode	
Arrow Keys	*Move right, left, up and down.*
Return Key	*Mark a vertex, start or end a line segment.*
Spacebar	*Lift pen. (Return puts it down)*
Delete	*Remove last line segment.*
Escape	*Exit drawing mode.*

Working with the Mirrors Reflect exercises develops in children the ability to define and recognize reflections. The task involves transferring an image from one side to the opposite and helps to develop laterality, an important psychomotor skill. To complete the exercise the student must visualize the reflection of a given figure. The child's drawing of the reflection is checked by the computer and any discrepancies are shown on the screen.

Teachers can use the program to help students understand symmetry and congruence. By completing the left side of drawings when given the right side of squares, rectangles and regular triangles and

trapezoids, students will discover that these figures have bilateral symmetry.

The program has several options which allow the teacher to use the program in a variety of ways:

Clear   The Clear option erases the screen but does not change the Mirror option.

Mirror   The Mirror option allows the type of reflection to be selected. The mirror can be placed on the vertical or horizontal axis or on both axes.

Sequence   The Sequence option refers to the order in which problems are presented. The program disk stores 30 mirror problems, 10 for each mirror setting. If the sequence is set to Select the student is given the opportunity to select the problems to be solved. When the setting is random the computer randomly selects the problems.

From the first menu follow these steps to solve a reflection problem:

1. Choose Options and then select Mirror to choose the type of mirror. Select Vertical for this first exercise.

2. Next choose sequence and then choose Select to set up the program so that you can select the exercise.

3. Choose Ok to leave the options menu and choose Begin.

4. The options Create or Solve appear on the menu bar. Since we want to solve one of the problems on the Hands-On Math Data Disk, choose Solve.

5. Next select the appropriate drive and make sure that the disk is inserted.

6. A list of challenges appears on the menu bar. Each challenge is numbered 1 through 10. For this example choose Challenge #1. The program will access the storage disk and redraw a figure. Your screen should look like this:

|||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||||

Draw Mode

```
+ + + + + + + + + + + + ┆+ + + + + + + + + + + +
+ + + + + + + + + + + + ┆+ + + + + + + + + + + +
+ + + + + + + + + + + + ┆+ + + + + + + + + + + +
+ + + + + + + + + + + + ┆+ + + +╱╲+ + + + + + +
+ + + + + + + + + + + + ┆+ + + ╱  ╲+ + + + + +
+ + + + + + + + + + + + ┆+ +╱    │+ + + +
+ + + + + + + + + + + + ┼─┤+ │    │+ + + +
+ + + + + + + + + + + + ┆+ +│    │+ + + +
+ + + + + + + + + + + + ┆+ +│    │+ + + +
+ + + + + + + + + + + + ┆+ +│    │+ + + +
+ + + + + + + + + + + + ┆+ +└────┘+ + + +
+ + + + + + + + + + + + ┆+ + + + + + + + + + + +
+ + + + + + + + + + + + ┆+ + + + + + + + + + + +
+ + + + + + + + + + + + ┆+ + + + + + + + + + + +
+ + + + + + + + + + + + ┆+ + + + + + + + + + + +
```
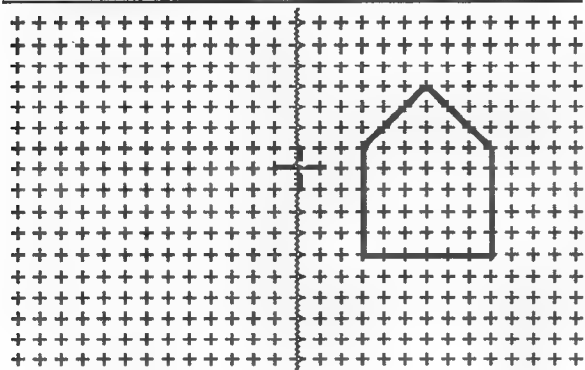
7. The program is now in the draw mode. The arrows, return, spacebar, delete and escape keys function as described above. The object is to draw a reflection of this figure.

8. Press escape after making the drawing and the menu bar shows these options: Check, New_Problem, Cancel

9. Choose Check and the program will draw the solution to the reflection problem in orange. Compare your solution with the solution drawn by the computer.

Solving a reflection problem can be very challenging to young students. In order to correctly draw the reflection of an object, the student must mentally flip the object. Teachers or students can use this program to create reflection problems. In creating a reflection problem the drawing point is moved about the grid and return is pressed to mark the vertices of the figure. Follow these steps to create new reflection problems:

1. Choose Begin and Create to start a drawing.

2. The menu bar will give the choices Vertical, Horizontal and Both for the types of mirrors to be used in creating the drawing. For this example select Vertical.

3. Draw the figure shown below.

Draw Mode.   Pen Down

4. Press escape to exit the Draw Mode. The menu bar now shows the choices Drive 1 or Drive 2. Select the appropriate drive for your system, insert the storage disk if necessary and press the return key.

5. The menu bar shows a list of problem numbers, Challenge #1, 2, 3, etc. Select the challenge number and press return.

Mirrors Symmetry
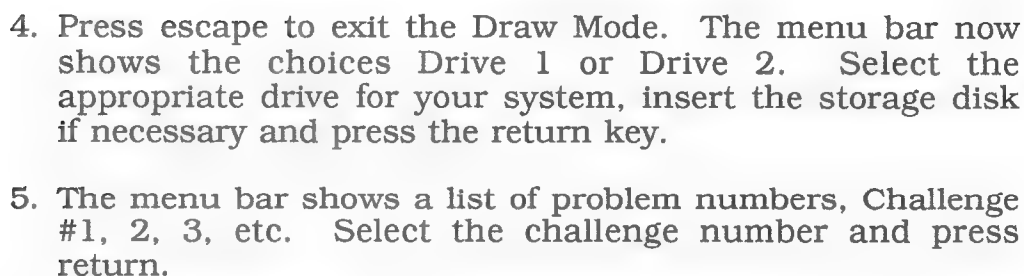
Objectives: Define and recognize a particular geometric transformation: the reflection. Given a figure, locate a bilateral axis of symmetry. Define, name and draw line segments, angles, planes, vertices, polygons, horizontal and vertical lines, perpendicular lines, intersecting lines, squares, rectangles, parallelograms, trapezoids and triangles.

The Mirrors Symmetry exercise allows a teacher or student to create reflection problems as in the Mirrors Reflect program. The difference here is that the task is to locate the axis of symmetry rather than draw the reflection. Once a problem has been created it can be solved and checked by the computer. An example of a symmetry problem is shown below where the object is to place the appropriate mirror on the axis of symmetry.

Position the axis of symmetry.

The Option menu has only one option.

Clear   The Clear option erases the screen.

Solve: From the first menu follow these steps to solve a symmetry problem:

1. Choose Begin and then select Solve from the menu bar.

2. Select the appropriate disk drive for the storage disk and insert the disk. Press enter when ready.

3. Select the Challenge by number. For this example choose Challenge #1. The problem is drawn on the screen.

4. Study the problem and decide which is the appropriate type of symmetry: vertical, horizontal or both.

5. Use the arrow keys to move the flashing lines to show the line(s) of symmetry. Press Return.

6. Choose Check and the computer will draw the correct lines of symmetry in orange on the screen. Select New Problem to get a new problem or Cancel to return to the first menu.
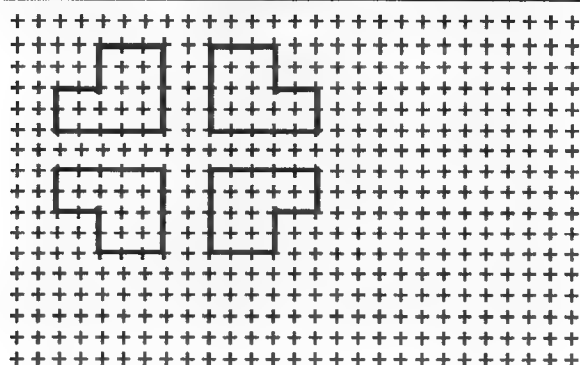
Create: Follow these steps to create a symmetry problem:

1. Choose Begin and Create from the menu. The menu bar will give the choices Vertical, Horizontal and Both for the types of mirrors to be used in creating the drawing. For this example select Vertical. The vertical axis will appear on the screen. Use the arrow keys to place the line as shown below.

2. Press the return key and the line becomes fixed on the screen.

3. Flashing cross hairs appear marking the drawing point. Move the cross hairs using the arrow keys and draw the figure shown here. The spacebar is used to lift the drawing point.

5. Press escape to exit the Draw Mode. The menu bar now shows the choices Drive 1 or Drive 2. Select the appropriate drive for your system, insert the storage disk if necessary and press the return key.

6. The menu bar shows a list of problem numbers, Challenge #1, 2, 3, etc. Select the challenge number and press return.

The drawing is now stored on the storage disk using the number selected in step six. The drawing can be recalled using the steps described in the Solve section.

Attribute Blocks Bridges

Objectives: Define and recognize a logical sequence based on the size, shape or color of objects.

The Attributes Bridges program develops logical thinking skills and helps children determine critical differences. The program uses the attribute block pieces to develop these skills. When the program starts, a bridge is drawn on the screen and two attribute blocks fall from the sky. Dots mark five positions along the bridge spanning from one attribute block to the other. The object is to place pieces along the bridge so that there is only one difference between each piece. The Options menu allows the number of differences to be switched. The choices are 1, 2 or 3.

After choosing Begin, the first of the five positions will be flashing. The position of the flashing dot can be moved to the right or left with the arrow keys. Move the flashing dot to a desired position and press return. An array of attribute block pieces appears on the screen. Move the indicator to select a piece using the arrow keys and press return. The piece appears on the bridge. Continue playing pieces until all five positions are filled and there is only one difference between neighboring pieces. Press escape and the menu bar gives the choices: Check, Edit or Cancel.

Choose Check to test the answer or edit to revise the solution. If Cancel is selected the program returns to the first menu.

Attribute Blocks Grids

Objectives: Solve logical problems using deductive reasoning.

The Attribute Blocks Grids program uses a 4X4 grid of rectangular cells as a gameboard. The computer program generates a logic problem that is solved using deductive reasoning. The computer randomly assigns rules to each column and row of the grid. The object of the exercise is to determine the rules and completely fill in the grid with attribute blocks according to the rules assigned by the computer.

The Option menu allows the user to clear the gameboard and to set the number of attributes used in the exercise. The default is for just color and shape to be used. To make the game more challenging the teacher or student may use the Options menu to set the attributes used in the exercise to color, shape and size.

After Begin has been selected one of the sixteen cells begins to flash. Using the arrow keys the selected cell can be changed. To play the game, move the indicator to a cell and press return. An array of attribute blocks is shown on the screen. (Note when only two attributes, color and shape, are being used only the first four columns of the array are active.) Move the indicator to the desired piece and press return. If the piece has the correct attributes it will appear in the selected cell. If it is not correct the computer will give an explanation of why it is incorrect. Continue trying pieces until a piece is accepted. Use deduction and the clues given by the computer to determine which pieces are correct.

When all of the cells have been filled a final score is given. The score is the number of incorrect pieces attempted. The lower the score the better. Using logical thinking the pattern can be discovered after only a few of the pieces have been played. Students will want to improve their score by strategically selecting the pieces and cells attempted.

Attribute Blocks Toolkit

Objectives: Solve logical problems using deductive reasoning. Recognize congruent figures. Complete patterns based on shape, color and size.

The Attribute Blocks Toolkit allows teachers and students to construct screens that pose logic problems. For example, the screen shown below presents an incomplete pattern made from attribute blocks. The task for the student is to select the appropriate pieces to complete the pattern.
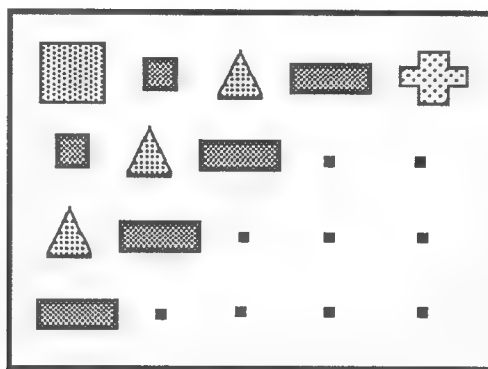
A variety of special shapes are available in the Attribute Blocks Toolkit. These shapes can be used to set up screens that represent logic problems. The screen shown above uses dots to show where the pieces are to be placed. The problem screens that are created can be stored on a disk and recalled for later use. The special shapes and the storage and retrieval functions are available from the Options menu which offers these choices:

Clear   The Clear option erases the screen.

Load   The Load option is used to retrieve a previously defined problem screen from the storage disk. The Hands-On Math: Volume II Data Disk contains 10 problem screens.

Save   The Save option is used to store problem screens on a storage disk. Remember: The Hands-On Math: Volume II Data Disk contains 10 problem screens. You may wish to store new screens on a different disk.

             Note: When using Load and Save the program prompts the user to select the appropriate drive and filename. The choices for drives are #1 and

#2. The filenames are Challenge #1 through #10.

Special

The Special option allows the user to select and place special objects on the screen. The special objects are positioned using the arrow keys. If an object has already been placed in a given position, the first time the user presses return the object at that position is removed. The second time return is pressed the new piece is placed in the position. The special objects are used to create activity screens. Complete directions for the activity screens are found in the activity pages section of this manual. The special objects are:

Arrow - Can be used to show the direction or flow of the logic in an arrangement of attribute blocks. The arrow can be rotated by pressing 'R' while it is selected. Load Challenge #2 to see an example. The object of this challenge is to play pieces with one difference and to follow the path indicated by the arrows. Challenge #9 shows an example of a different path made with arrows.

Start and Stop - The start and stop symbols can be used to mark the beginning and ending points of a trail. Challenge #3 shows an example. Place pieces along the trail that have two differences.

Bar - The bar can be used in making logic puzzles. Challenge #4 uses the bar with the box to make a puzzle where the object is to find two pieces that can be placed so that the differences between each piece is two. Challenge #7 shows another use of the bar and box. In the challenge the student puts any block in the box and then places pieces below the bar that differ in exactly one way. Challenge #10 shows a challenging logic problem where the student starts with different pieces and ends with the same piece.

Dot - Use the dot to show where the student should place a piece. Challenge #5 shows an example. Six dots are place on the screen. The object is to place six attribute blocks on the dots

so that each block differs from its horizontal and vertical neighbors in exactly one way.

Box - Use the box to indicate cells where the student should place a piece. Challenge #6 demonstrates a type of problem where the object is to place pieces along the I-shape so that each piece differs in two ways from its neighbor. Challenge #8 shows a different way to use the box. Here the challenge is to complete the square by finding pieces to place in the empty cells so that all the pieces differ in three ways from their neighbors.

Limit

The Limit option controls whether pieces can be used more than once. The two settings are 'On' and 'Off'. For some exercises, for example making patterns, teachers will want students to be able to repeatedly use pieces and so should set the Limit option to 'Off'. For activities where students must find pieces that differ in a specified number of ways teachers will want the limit 'On'.

Base Ten Blocks Place Value

Objectives: State the number when given a representation of a number using base ten blocks. Represent numbers using base ten blocks.

The Base Ten Blocks Place Value exercise challenges the student to enter the correct number given a representation of the number using base ten blocks. The options available in the program allow teachers to individualize each student's experience.

Clear

The Clear option erases the screen but leaves the Goal and Zero setting unchanged.

Goal

The Goal option allows the number of problems required to complete the exercise to be set.

Zero

The Zero option controls whether numbers are generated with zeros. If the zero option is 'Off' at least one block for each place value position will be displayed. When the zero option is 'On' it is possible for the correct answers to contain zeros as place holders. The default is for the zero option to be 'On'.

To complete the exercise choose Begin from the first menu. The program will display a set of blocks and the question, "What is the value of these blocks?" will appear on the menu bar. Press any key to continue and the input prompt will appear. Enter the answer and press return. For the example shown below the student would enter: 1,434. If the answer is correct the computer prints the word 'Correct!' and the current score. When an incorrect answer is given the student is given a second chance. If the problem is answered incorrectly a second time the correct answer is given. The object is to reach the goal selected using the option menu. The default goal is 10 problems. To quit before completing the exercise press return when asked to enter an answer.
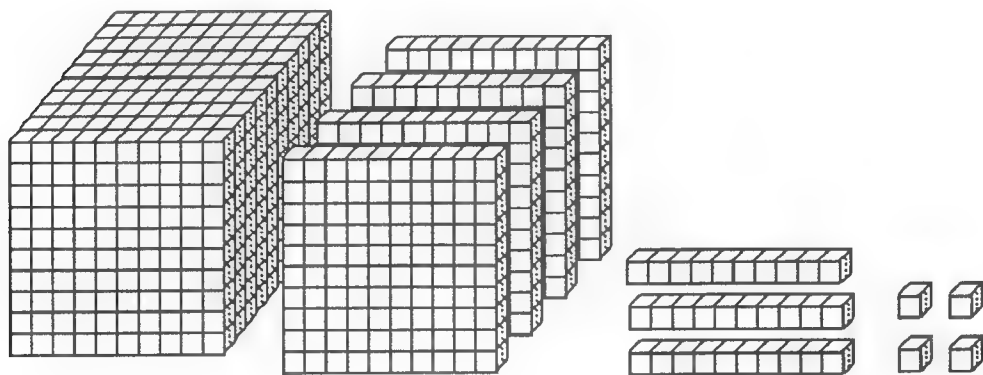
Answers to Activity Pages

Page 1: Part 1: The groups are equal.
         Part 2: Answers will vary.

Page 2: Answers will vary.

Page 3: Answers will vary.

Page 4: Answers will vary.

Page 5: Answers will vary. Possible answers:

         0 + 6 = 6   6 + 0 = 6
         1 + 5 = 6   5 + 1 = 6
         2 + 4 = 6   4 + 2 = 6
         3 + 3 = 6

         0 + 8 = 8   8 + 0 = 8
         1 + 7 = 8   7 + 1 = 8
         2 + 6 = 8   6 + 2 = 8
         3 + 5 = 8   5 + 3 = 8
         4 + 4 = 8

         0 + 5 = 5   5 + 0 = 5
         1 + 4 = 5   4 + 1 = 5
         2 + 3 = 5   3 + 2 = 5

Page 6: Answers will vary. Possible answers:

         0 + 12 = 12   12 + 0 = 12
         1 + 11 = 12   11 + 1 = 12
         2 + 10 = 12   10 + 2 = 12
         3 + 9 = 12   9 + 3 = 12
         4 + 8 = 12   8 + 4 = 12
         5 + 7 = 12   7 + 5 = 12
         6 + 6 = 12

         0 + 10 = 10   10 + 0 = 10
         1 + 9 = 10   9 + 1 = 10
         2 + 8 = 10   8 + 2 = 10
         3 + 7 = 10   7 + 3 = 10
         4 + 6 = 10   6 + 4 = 10
         5 + 5 = 10

         0 + 4 = 4   4 + 0 = 4
         1 + 3 = 4   3 + 1 = 4
         2 + 2 = 4

$$0 + 3 = 3 \qquad 3 + 0 = 3$$
$$1 + 2 = 3 \qquad 2 + 1 = 3$$

$$0 + 7 = 7 \qquad 7 + 0 = 7$$
$$1 + 6 = 7 \qquad 6 + 1 = 7$$
$$2 + 5 = 7 \qquad 5 + 2 = 7$$
$$3 + 4 = 7 \qquad 4 + 3 = 7$$

$$0 + 2 = 2 \qquad 2 + 0 = 2$$
$$1 + 1 = 2$$

$$0 + 11 = 11 \qquad 11 + 0 = 11$$
$$1 + 10 = 11 \qquad 10 + 1 = 11$$
$$2 + 9 = 11 \qquad 9 + 2 = 11$$
$$3 + 8 = 11 \qquad 8 + 3 = 11$$
$$4 + 7 = 11 \qquad 7 + 4 = 11$$
$$5 + 6 = 11 \qquad 6 + 5 = 11$$

Page 7: $8 - 5 = 3 \qquad 6 - 2 = 4$
                       $7 - 3 = 4 \qquad 9 - 4 = 5$
                       $4 - 3 = 1 \qquad 8 - 6 = 2$

Page 8:   Answers will vary.

Page 9: $1/2 \text{ of } 6 = 3 \qquad 1/2 \text{ of } 10 = 5$
                       $1/5 \text{ of } 5 = 1 \qquad 1/2 \text{ of } 4 = 2$
                       $2/3 \text{ of } 6 = 4 \qquad 3/5 \text{ of } 10 = 6$

Page 10:   Part 1: Answers will vary.
                     Part 2: Answers will vary.
                     Part 3: 1/2.

Page 11:   Part 1: 2 green, 2 violet, or 1 green and 1 violet.
                     Part 2: Answers will vary.
                     Part 3: 1/2.

Page 12:   Part 1: Answers will vary.
                     Part 2:   2   4
                                    4   5
                                    3   1

Page 13:   Answers will vary.

Page 14:   Answers will vary.

Page 15: Answers will vary. Possible answers:

      0 + 6 = 6   6 + 0 = 6
      1 + 5 = 6   5 + 1 = 6
      2 + 4 = 6   4 + 2 = 6
      3 + 3 = 6

      0 + 4 = 4   4 + 0 = 4
      1 + 3 = 4   3 + 1 = 4
      2 + 2 = 4

      0 + 5 = 5   5 + 0 = 5
      1 + 4 = 5   4 + 1 = 5
      2 + 3 = 5   3 + 2 = 5

Page 16: Answers will vary. Possible answers:

      0 + 8 = 8   8 + 0 = 8
      1 + 7 = 8   7 + 1 = 8
      2 + 6 = 8   6 + 2 = 8
      3 + 5 = 8   5 + 3 = 8
      4 + 4 = 8

      0 + 9 = 9   9 + 0 = 9
      1 + 8 = 9   8 + 1 = 9
      2 + 7 = 9   7 + 2 = 9
      3 + 6 = 9   6 + 3 = 9
      4 + 5 = 9   5 + 4 = 9

      0 + 7 = 7   7 + 0 = 7
      1 + 6 = 7   6 + 1 = 7
      2 + 5 = 7   5 + 2 = 7
      3 + 4 = 7   4 + 3 = 7

Page 17: 3 + 4 = 7
      5 + 2 = 7
      3 + 3 = 6
      3 + 5 = 8
      6 + 3 = 9

Page 18: 3 + 7 = 10
      4 + 6 = 10
      1 + 9 = 10
      5 + 5 = 10
      8 + 2 = 10
      2 + 8 = 10

Page 19:   2 x 4 = 8   3 x 6 = 18
                3 x 7 = 21   4 x 4 = 16

Page 20:   2 x 5 = 10   3 x 6 = 18
                8 x 7 = 56   6 x 6 = 36
                3 x 4 = 12   5 x 5 = 25
                8 x 5 = 40   4 x 6 = 24

Page 21:   10 ÷ 5 = 2   9 ÷ 3 = 3
                8 ÷ 2 = 4   12 ÷ 3 = 4

Page 22:   6 ÷ 2 = 3   15 ÷ 5 = 3
                12 ÷ 6 = 2   18 ÷ 9 = 2
                20 ÷ 5 = 4   14 ÷ 2 = 7
                16 ÷ 8 = 3   20 ÷ 2 = 10

Page 23:   6 green   4 orange
                6 violet   12 blue
                5 green   8 orange
                10 violet   4 blue

                6 green   3 orange
                10 violet   15 blue

Page 24:   Answers will vary.

Page 25:   Answers will vary.

Page 26:   Part 1: Left side should be a mirror image of right side.
                Part 2: Patterns will continue as shown.

Page 27:   Answers will vary.

Page 28:   Answers will vary.

Page 29:   Answers will vary.

Page 30:   Part 1: Answers will vary.
                Part 2:   10 = 2 x 5
                            12 = 3 x 4

Page 31:   Part 1:   A. 4,222   C. 3,332
                            B. 2,532   D. 2,054
                Part 2: Answers will vary.

Page 32:   Part 1: H, I, M, O, T, U, V, W, X, Y
                Part 2: C, D, E, H, I, O, X

Page 33: Parallelogram
Triangle
Trapezoid
Triangle
Hexagon
Octagon

Page 34: Sketches should be mirror images of figures on right.

Page 35: Sketches should be mirror images of figures on right.

Page 36: Correct answers are shown on computer.

Page 37: Correct answers are shown on computer.

Page 38: Lines should be drawn from the head to the tail along the back of each animal.

Page 39: Part 1: Answers will vary.
Part 2: Answers will vary.
Part 3: Green Square, Violet Cross, Green Square, Violet Cross
Part 4: Green Square, Violet Cross

Page 40: Answers will vary.

Page 41: Answers will vary.

Page 42: Answers will vary.

Page 43: Boxes should be checked.

Page 44: Part 1: Boxes should be checked.
Part 2:
1. hundreds	2. tens
3. ones	4. tens
5. thousands	6. tens
7. hundreds	8. tens
9. tens	10. hundreds

Part 3: 2,356
3,075
4,226
8,029

Page 45: 1,547 one thousand five hundred forty-seven
2,227 two thousand two hundred twenty-seven
2,405 two thousand four hundred five

Page 46:   1,573   one thousand five hundred seventy-three
           1,365   one thousand three hundred sixty-five
           1,549   one thousand five hundred forty-nine
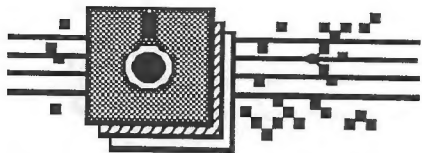
Activity Pages

The following pages may be reproduced for classroom use. These pages serve as a guide for students who are using the Hands-On Math program.